Module - 1

1.1. Introduction to mechanics.

Engineering mechanics is the physical science which describes and predicts the conditions of rest or of motion of bodies under the action of forces.It is a physical science because it deals with the study of physical phenomena.The subject is subdivided into statics and dynamics. Statics deals with the action of forces on bodies which are at rest and remain at rest. Dynamics deals with the motion of bodies under the action of forces. It has two distinct parts - kinematics and kinetics. Kinematics is the study of motion of bodies without reference to the forces which either cause the motion or are generated as a result of the motion. Kinetics is the study of the relationship between motion and the corresponding forces which cause or accompany the motion.

Modern engineering practice demands a high level of analytical capability and the study of engineering mechanics is to build a foundation of analytical capability for the solution of a variety of engineering problems. In fact no other physical science plays a greater role in engineering than does mechanics. In engineering mechanics we learn to construct and solve mathematical models which describe the effects of forces and motion on a variety of structures and mechanisms that are concern to engineers.

Basic concepts.

The basic concepts used in mechanics are space, time, mass and force. These concepts cannot be truly defined, they should be accepted on the basis of our intuition and experience and are used as a mental frame of reference for our study of mechanics.

The concept of space is associated with motion of the position of a point. The position of the point may be defined by three lengths measured from certain reference lines, in three given directions. These lengths are known as the co-ordinates of the point.

The concept for ordering the flow of events is the time. It is the measure of succession of events.The rotation of earth gives rise to an event that serves as a good measure of time, the day. For engineering applications we need smaller units and thus, generally, we tie events to the second, which is an action repeatable 86400 times a day.

The concept of mass is used to characterise and compare bodies on the basis of the response to a mechanical disturbance. Two bodies of the same mass will be attracted by the earth in the same manner. They will also offer the same resistance to a change in translational motion. Thus mass is the quantitative measure of the resistance to change in motion of a body.

Force represents the action of one body on another. It may be exerted by actual contact or at a distance as in the case of gravitational force and magnetic force. A force is characterised by its point of application, magnitude and direction.

A particle is a body of negligible dimensions and hence may be assumed to occupy a single point in space. When the dimensions of a body are irrelevant to the description of its motion or the action of forces on it, the body may be treated as a particle. For example, even an aeroplane can be treated as a particle for the description of its flight path

A rigid body is a body which does not deform when subjected to external forces. However, bodies are never absolutely rigid and hence deform under the forces to which they are subjected. When the deformation is negligibly small compared with the overall dimensions of the body, the body can be treated as rigid.

Fundamental principles.

The study of elementary mechanics is based on six fundamental principles.

1. The parallelogram law of forces.

If two forces acting simultaneously at a point are represented in magnitude and direction by the adjacent sides of a parallelogram then the diagonal of the parallelogram passing through their point of intersection represents the resultant of the two forces in both magnitude and direction.

2. Principle of transmissibility.

The condition of equilibrium or of motion of a rigid body will remain unchanged if the point of application of a force acting on the rigid body is transmitted to any other point on the line of action of the force.

3. Newton's first law.

Every body continues in its state of rest or of uniform motion,in a straight line, unless it is acted upon by some external force.

4. Newton's second law.

The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.

5. Newton's third law.

To every action there is an equal and opposite reaction.

6. Newton's law of gravitation.

Two particles are attracted towards each other along the line connecting them with a force whose magnitude is proportional to the product of their masses and inversely proportional to the square of the distance between them.

$F \propto \frac{m_1 m_2}{r^2}$, F is the force of attraction , m_1 and m_2 are the masses of particles and r is the distance between the particles.

$F = \frac{G m_1 m_2}{r^2}$ where G is the universal constant called constant of gravitation.

When a particle of mass m lies on the surface of the earth, the force exerted by the earth is the weight of the particle. Weight of the particle $W = F = \frac{GMm}{R^2} = mg$ where $g = \frac{GM}{R^2}$, is the accelaration due to gravity. M is the mass of the earth and R is the radius of the earth. The value of g depends on the distance of particle from the centre of earth and latitude. For most of the engineering calculations g is calculated at sea level and at a latitude of 45^0. Here $g = 9.80665\ m/s^2$, however, for calculations the value of $g = 9.81\ m/s^2$ can be used.

System of units.

The four fundamental quantities of mechanics, mass, length, time and force are related by the fundamental principle,

$$F = m \times a$$

$$\text{Force} = \text{mass} \times \frac{\text{length}}{\text{time}^2}$$

The units of any three of the above fundamental quantities are defined arbitrarily and these three units are referred to as basic units. The unit of the fourth quantity must be chosen in accordance with the fundamental principle, $F = m \times a$, and is referred to as derived unit. The units of all other physical quantities are derived from the basic units and the unit of the fourth quantity. The units selected in this way are said to form a system of units.

System International Units (SI Units).

In this system of unit, the basic units are the units of length , mass, and time and are respectively, metre (m), kilogram (kg) and second (s). The unit of force is a derived unit.

Force = mass x length/ $(\text{time})^2$

Unit of force = unit of mass x unit of length / $(\text{unit of time})^2$ = $kg \frac{m}{s^2}$

Thus the derived unit of force is $kg\frac{m}{s^2}$. 1 kg m /s² is defined as one newton (N) and it is the force which gives an acceleration of $1 m/s^2$ to a mass of 1kg.

$$1\ N = 1\ kg\ \ m/s^2$$

Like any other force, the weight of a body should be expressed in newton. The weight of a body of mass m kg is m x g , where g is the acceleration due to gravity.

Weight of m kg of mass = W = mg

Weight of 1 kg of mass = $1\ kg. \times\ 9.81 m/s^2 = 9.81\ kg.\ m/s^2 = 9.81\ N$

Various SI units used in mechanics are given in table 1.1.

Table 1.1

Quantity	Unit	SI symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Area	$metre^2$	m^2
Volume	$metre^3$	m^3
Linear velocity	metre/second	m/s
Angular velocity	radian/second	rad/s
Linear acceleration	metre /$second^2$	m/s^2
Angular acceleration	radian/$second^2$	rad/s^2
Density	kilogram/$metre^3$	kg/m^3
Area moment of inertia	$metre^4$	m^4
Mass moment of inertia	kilogram - $metre^2$	kg -m^2
Frequency	hertz	Hz
Force	newton	N
Moment of force	newton - metre	N-m
Pressure	pascal	Pa
Work, energy	joule	J
Power	watt	W

1.2 Statics

Statics is the study of particles and rigid bodies at rest. It concerns the equilibrium of particles and bodies under the action of forces.

1.3 Basic Principles of statics.

The major basic principles involved in statics are;

1. Newton's first law
2. Parallelogram law of forces
3. Principle of transmissibility of force.

Other basic principles of statics are:

1. Equilibrium law
2. Principle of superposition
3. Law of action and reaction.

1.4 Newton's first law.

It states that every body continues in its state of rest or of uniform motion in a straight line unless it is compelled by a force to change that state.

A body which is at rest will continue to be at rest or a body which is moving with a constant speed in a particular direction will continue moving with the same speed in the same direction. The state of rest or that of uniform motion will change only when it is acted upon by an unbalanced external force. The property of the body which resists any effort to change its state of rest or of uniform motion is known as inertia of the body. Thus the Newton's first law brings out the concept of inertia and hence it is also known as law of inertia.

1.5 Parallelogram law:

Slevinius (1548 - 1620) was the first to demonstrate that two forces could be combined by representing them by directed lines to some suitable scale and then forming a parallelogram with the two forces as adjacent sides. The law states that, "If two forces acting simultaneously at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces".

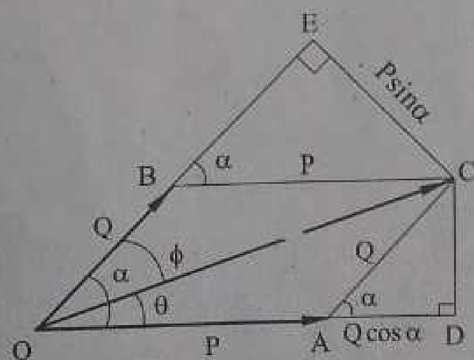


Fig. 1.1

Let two forces P and Q act at a point O as shown in Fig. 1.1. These forces P and Q are represented in magnitude and direction by the vectors OA and OB respectively.

Let the angle between the two forces be α. Draw the parallelogram with OA and OB as adjacent sides. The diagonal OC, according to the parallelogram law of forces, represents the magnitude and direction of the resultant R of the forces P and Q. From C, draw CD perpendicular to OA produced. In the triangle OCD,

$$OC^2 = OD^2 + DC^2$$

$$= (P + Q\cos\alpha)^2 + (Q\sin\alpha)^2$$

$$= P^2 + Q^2\cos^2\alpha + 2\,PQ\cos\alpha + Q^2\sin^2\alpha$$

$$= P^2 + Q^2\cos^2\alpha + Q^2\sin^2\alpha + 2PQ\cos\alpha$$

$$= P^2 + Q^2[\cos^2\alpha + \sin^2\alpha] + 2PQ\cos\alpha$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

Therefore the resultant of P and Q,

$$R = \sqrt{P^2 + Q^2 + 2\,PQ\cos\alpha}$$

Let θ be the inclination of resultant with force P, then,

$$\tan\theta = \frac{CD}{OD} = \frac{Q\sin\alpha}{P+Q\cos\alpha} = \frac{\sin\alpha}{\frac{P}{Q}+\cos\alpha}$$

If ϕ is the inclination of resultant with force Q, then,

$$\phi = \alpha - \theta,$$

$$\tan\phi = \frac{CE}{OE} = \frac{P\sin\alpha}{Q+P\cos\alpha} = \frac{\sin\alpha}{\cos\alpha+\frac{Q}{P}}$$

$$\therefore \phi = \tan^{-1}\frac{\sin\alpha}{\cos\alpha + \frac{Q}{P}}$$

Particular cases :

(i) When $\alpha = 0$

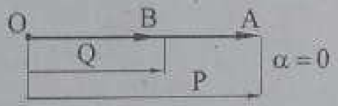


Fig. 1.2

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$= \sqrt{P^2 + Q^2 + 2PQ\cos 0}$$

$$= \sqrt{P^2 + Q^2 + 2PQ}$$

$$= \sqrt{(P + Q)^2}$$

$$R = P + Q$$

(ii) when $\alpha = 90^0$

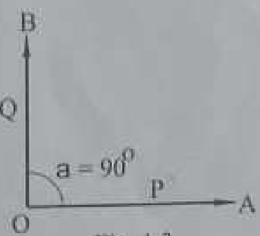


Fig. 1.3

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$= \sqrt{P^2 + Q^2 + 2PQ\cos 90}$$

$$R = \sqrt{P^2 + Q^2}$$

(iii) when $\alpha = 180^0$

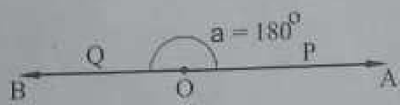


Fig. 1.4

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$= \sqrt{P^2 + Q^2 + 2PQ\cos 180}$$

$$= \sqrt{P^2 + Q^2 + 2\,PQ\,(-1)}$$

$$= \sqrt{P^2 + Q^2 - 2PQ}$$

$$= \sqrt{(P - Q)^2}$$

$$R = P - Q$$

(iv) when $Q = P$

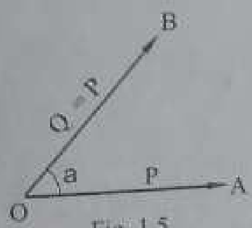


Fig. 1.5

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

$$= P^2 + P^2 + 2.\,P.P.\cos\alpha$$

$$= 2P^2 + 2P^2\cos\alpha$$

$$= 2P^2\,[\,1 + \cos\alpha]$$

$$= 2P^2 \times 2\cos^2\frac{\alpha}{2}$$

$$= 4P^2\cos^2\frac{\alpha}{2}$$

$$R = 2P\cos\frac{\alpha}{2}$$

$$\tan\theta = \frac{\sin\alpha}{\cos\alpha + \frac{P}{Q}}$$

$$= \frac{\sin\alpha}{1+\cos\alpha}$$

$$= \frac{2\sin\frac{\alpha}{2}.\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}}$$

$$= \tan\frac{\alpha}{2}$$

$\therefore \theta = \frac{\alpha}{2}$ ie, when Q = P, the resultant bisects the angle between the forces

Example 1.1 [KTU Jan 2017]

The greatest and least resultants of two forces F_1 and F_2 are 17N and 3N respectively. Determine the angle between them when their resultant is 149N.

Solution

Given

$$F_1 + F_2 = 17N$$

$$F_1 - F_2 = 3N$$

To calculate, α when $R = \sqrt{149}$

$$F_1 + F_2 = 17$$

$$F_1 - F_2 = 3$$

Adding the above equations,

$$2F_1 = 20$$

$$F_1 = 10\ N$$

$$F_2 = 17 - 10 = 7N$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\alpha}$$

$$149 = 102 + 72 + 2\times10\times7\times\cos\alpha$$

$$2\times10\times7\cos\alpha = 0$$

$$\alpha = 90$$

Example 1.2

Two forces P and Q of magnitude 25N and 10N are acting at a point. The forces P and Q make angle 15° and 45°, measured counter clockwise with the horizontal. Determine the resultant in magnitude and direction.

Solution :

$P = 25$ N

$Q = 10$ N

$\alpha = 45 - 15 = 30^0$

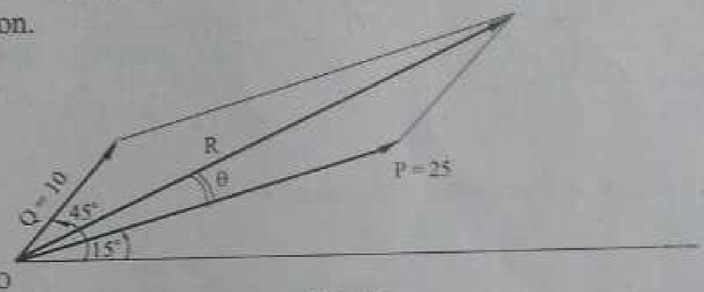


Fig. 1.6

$$\text{Resultant, } R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$= \sqrt{25^2 + 10^2 + 2\times25\times10\times\cos 30}$$

$$R = 34.03 \text{ N.}$$

The inclination of resultant force with the direction of force P,

$$\theta = \tan^{-1}\frac{\sin\alpha}{\cos\alpha + \frac{P}{Q}}$$

$$\theta = \tan^{-1}\frac{\sin 30}{\cos 30 + \frac{25}{10}}$$

$$\theta = 8.45^0$$

Inclination of resultant with horizontal is $15^0 + \theta$

$$= 15^0 + 8.45^0$$

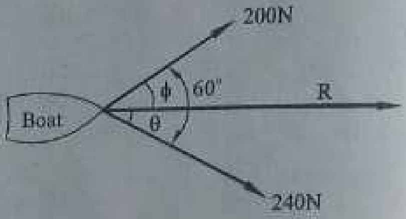
$$= 23.45^0$$

Example. 1.3

A boat is moved uniformly by pulling with forces 200N and 240N acting at an angle 60^0 as shown in Fig.1.7. Determine the magnitude of the resultant pull on the boat and the angle made by the resultant with forces 200N and 240N.

Solution

Given.



$P = 240\ N;\quad Q = 200\ N;\quad \alpha = 60^0$

$$\text{Resultant } R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

Fig. 1.7

$$= \sqrt{240^2 + 200^2 + 2 \times 240 \times 200 \cos 60}$$

$$R = 381.58\ N$$

Inclination of resultant with force 240 N,

$$\tan\theta = \frac{\sin 60}{\cos 60 + (\frac{240}{200})}$$

$$= 0.5094$$

$$\theta = 27^0$$

Inclination of resultant with force 200 N,

$$\phi = \tan^{-1}\frac{\sin 60}{\cos 60 + \frac{200}{240}}$$

$$\phi = 33^0\ [\phi = 60 - \theta = 60 - 27 = 33^0]$$

Example 1.4

A boat is moved uniformly by pulling with forces P = 240N and Q = 200N. What must be the inclination of the resultant force with P and Q to have the resultant R = 400N as shown in Fig 1.8.

Solution

Given

$$P = 240\ N$$

$Q = 200$ N

$R = 400$ N

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

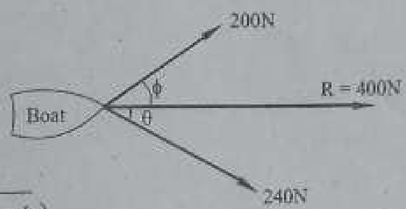


Fig. 1.8

$$400 = \sqrt{240^2 + 200^2 + 2 \times 240 \times 200\cos(\theta + \phi)}$$

$$\cos(\theta + \phi) = 0.65$$

$$\theta + \phi = 49.46^0$$

Inclination of resultant force with P,

$$\theta = \tan^{-1}\frac{\sin(\theta+\phi)}{\cos(\theta+\phi) + \frac{P}{Q}}$$

$$= \tan^{-1}\frac{\sin 49.46}{\cos 49.46 + \frac{240}{200}}$$

$$= 22.33^0.$$

Inclination of resultant with Q,

$$\phi = 49.46^0 - 22.33 = 27.13.$$

$$= 27.13^0$$

Example 1.5

The resultant of two forces when they act at an angle of 60° is 14N. When they act at right angles, their resultant is 12N. Determine the magnitude of the two forces.

Solution:

Case(i) $R = 14$ N, $\alpha = 60^0$

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

$$14^2 = P^2 + Q^2 + 2PQ.\cos 60^0$$

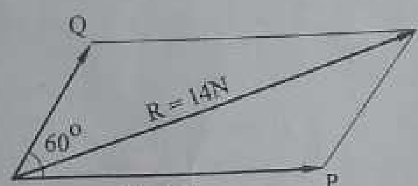


Fig. 1.9

$P^2 + Q^2 + PQ = 196$ -------------(i)

Case(ii) $R = 12$ N, $\alpha = 90^0$

$$R^2 = P^2 + Q^2$$

$12^2 = P^2 + Q^2$. Substituting this value of $P^2 + Q^2$ in eqn (i).

$$12^2 + PQ = 196$$

$$PQ = 52$$

$$(P+Q)^2 = P^2 + Q^2 + 2PQ$$

$$= 12^2 + 2 \times 52$$

$$P + Q = 15.75 \text{ N}$$

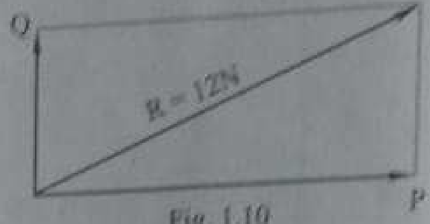


Fig. 1.10

$$(P - Q)^2 = P^2 + Q^2 - 2PQ = 144 - 2 \times 52 = 40$$

$$P - Q = 6.32 \text{ N}$$

$$P + Q = 15.75 \text{ N}$$

$$2P = 22.07 \text{ N}$$

$$P = 11.04 \text{ N}$$

$$Q = 15.75 - 11.04$$

$$= 4.71 \text{ N.}$$

1.6 Principle of transmissibility.

The principle of transmissibility states that the point of application of a force can be transmitted along its line of action without changing the effect of the force on any rigid body to which it is applied.

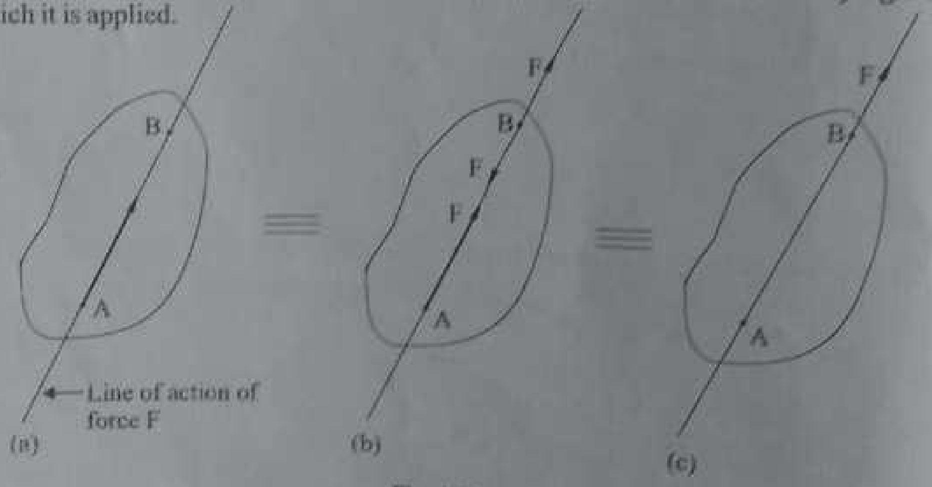


Fig. 1.11

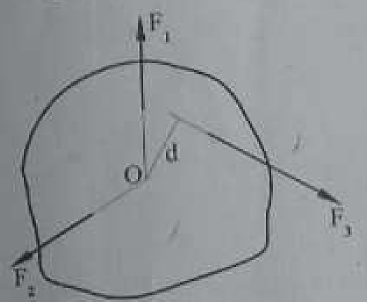
Consider a rigid body under the action of a force F applied at A and acting along AB as shown in Fig. 1.11. Two equal and opposite forces applied at B will not change the condition of the rigid body. Now the removal of force at A and the force at B which is opposite to the force at A will not change the condition of the rigid body. The condition of rigid body at Fig 1.11 (c) is same as that at Fig. 1.11 (a). This proves that trasmission of force F from its point of application at A to another point B which is in the line of action of force F does not change the condition of the rigid body.

1.7. Equilibrium laws.

Equilibrium of a body is the condition in which the resultant of all the forces and moments acting on the body is zero. For a two force body to be in equilibrium the forces must be equal in magnitude, opposite in direction and must be collinear. This is the required condition for $F = 0$ and $M = 0$.

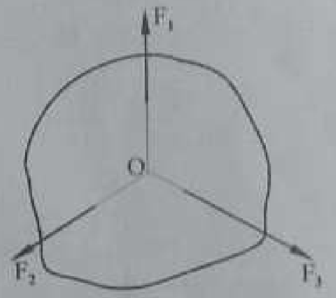
Fig. 1.12

For a three force body to be in equilibrium all the three forces must be concurrent forces. The line of action of the forces should meet at a point. This is the required condition for $M = 0$. If the three forces are not concurrent then one of the forces will exert a moment about the point of intersection of the other two forces. Refer Fig. 1.13. The magnitude of the three forces should be such that the vector sum of any two forces must be equal and opossite to the third force. This condition is required to satisfy $F = 0$. Refer Fig. 1.14. If this condition is satisfied then the three forces can be represented by the three sides of a triangle. Refer Fig. 1.15



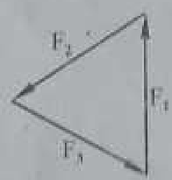
$M \neq 0$

Fig. 1.13



$M = 0$

Fig. 1.14



$F = 0$

Fig. 1.15

Law of triangle of forces.

If three coplanar forces acting at a point are in equilibrium then they can be represented in magnitude and direction by the side of a triangle taken in the same order. Fig. 1.16 shows the three forces F_1, F_2 and F_3 acting at a point O, keeping the point in equilibrium. Let the forces F_1 and F_2 be represented in magnitude and direction by OA and AB as shown in Fig. 1.17 then the closing side of the triangle, BO, represents the force F_3 in both magnitude and direction.

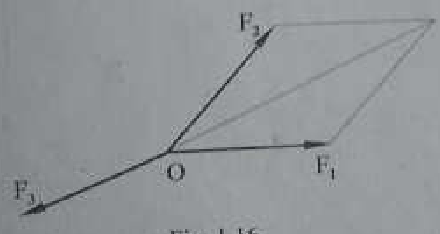


Fig. 1.16

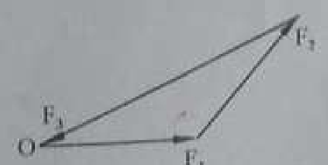


Fig. 1.17

Lami's theorem.

It is another equilibrium law which states that if three forces acting at a point are in equilibrium then each force is proportional to the sign of the angle between the other two forces. Consider three forces F_1 F_2 and F_3 acting at a point. Let the angle between F_1 and F_2 be γ, angle between F_1 and F_3 be β and angle between F_2 and F_3 be α as shown in Fig. 1.18. If the three forces are in equilibrium then according to Lami's theorem,

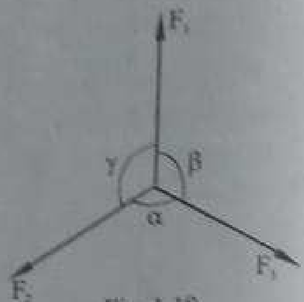


Fig. 1.18

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

Proof of Lami's theorem.

When three forces F_1, F_2 and F_3 acting at a point are in equilibrium, these forces can be represented by the three sides of a triangle OAB as shown in Fig. 1.19

Applying sine rule, $\frac{OA}{\sin(180-\alpha)} = \frac{AB}{\sin(180-\beta)} = \frac{BO}{\sin(180-\gamma)}$

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

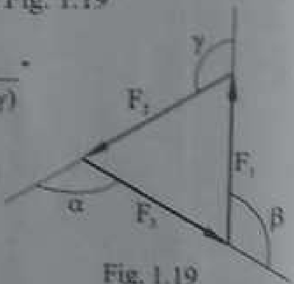


Fig. 1.19

Example 1.6.

An electric light fixture weighing 150 N hangs from a point C by two stay wires AC and BC as shown in Fig 1.20 Determine the tensions in the stay wires AC and BC using Lami's theorem.

Solution

Using Lami's theorem

$$\frac{150}{\sin 75} = \frac{T_{BC}}{\sin 150} = \frac{T_{AC}}{\sin 135}$$

$$T_{BC} = \frac{150 \times \sin 150}{\sin 75} = 77.65 \text{ N}$$

$$T_{AC} = \frac{150 \times \sin 135}{\sin 75} = 109.81 \text{ N}$$

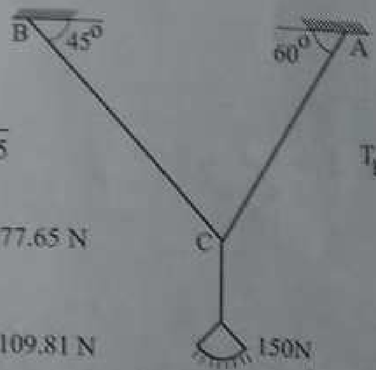


Fig. 1.20

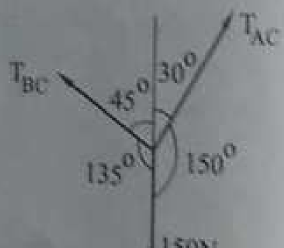


Fig. 1.21

Example 1.7.

Two cables AC and BC are tied together at the point C to support a load of 500 N at C. A and B are at a distance of 1.3m and are on the same horizontal plane. AC and BC are 1.2 m and 0.5m respectively. Find the tensions in the cables AC and BC.

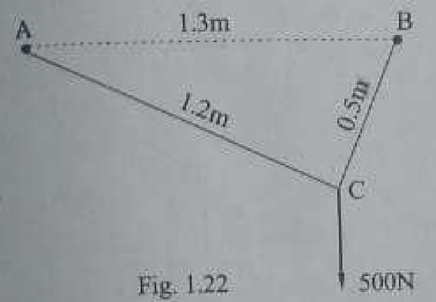


Fig. 1.22

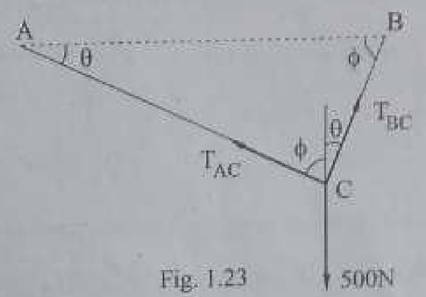


Fig. 1.23

Solution.

$$AC^2 + BC^2 = 1.2^2 + 0.5^2 = 1.69 = 1.3^2 = AB^2$$

Since $AC^2 + BC^2 = AB^2$, angle ACB = 90^0

$$\theta + \phi = 90^0$$

$$1.3 \cos\theta = 1.2$$

$$\theta = \cos^{-1}\frac{1.2}{1.3} = 22.62^0$$

$$\phi = 90 - \theta = 90 - 22.62 = 67.38^0$$

Applying Lami's theorem,

$$\frac{500}{\sin(\theta + \phi)} = \frac{T_{AC}}{\sin(180 - \theta)} = \frac{T_{BC}}{\sin(180 - \phi)}$$

$$\frac{500}{\sin 90} = \frac{T_{AC}}{\sin(180 - 22.62)} = \frac{T_{BC}}{\sin(180 - 67.38)}$$

$$T_{AC} = \frac{500\sin(180 - 22.62)}{\sin 90} = 192.31\text{ N}$$

$$T_{BC} = \frac{500\sin(180 - 67.38)}{\sin 90} = 461.54\text{ N}$$

Example 1.8.

A block P = 5kg and block Q of mass M kg are suspended through a chord which is in equilibrium as shown in Fig. 1.24. Determine the mass of the block Q.

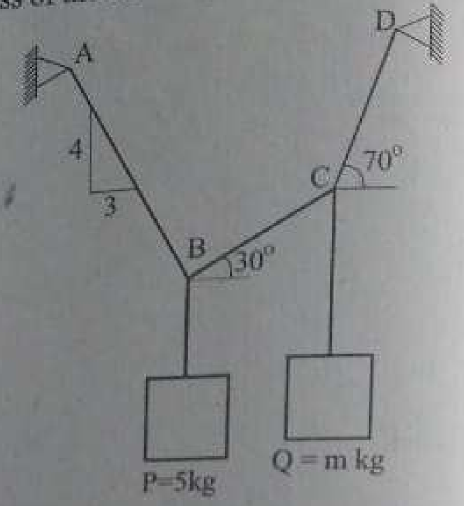


Fig. 1.24

Solution

Inclination of chord AB with horizontal is

$$\tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ.$$

$$\theta = 53.13^\circ$$

$$\angle ABC = 180 - (30 + 53.13) = 96.87$$

Point B is acted upon by 3 forces, tension in the chord T_{AB}, TBC and weight $W \times 9.81$.

Applying Lami's theorem,

$$\frac{5\times9.81}{\sin 96.87} = \frac{T_{AB}}{\sin 120} = \frac{T_{BC}}{\sin 143.13}$$

$$T_{AB} = \frac{5\times9.81}{\sin 96.87}\times\sin 120 = 42.79\text{N}$$

$$T_{BC} = \frac{5\times9.81}{\sin 96.87}\times\sin 143.13$$

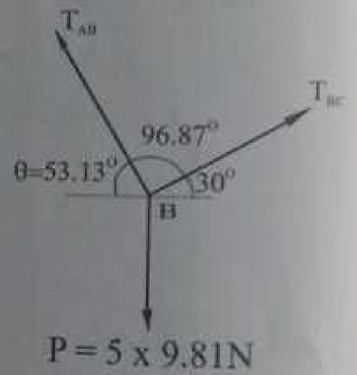


Fig. 1.25

Consider the equilibrium of point C.

Applying Lami's theorem,

$$\frac{5\times9.81}{\sin 96.87} = \frac{T_{AB}}{\sin 120} = \frac{T_{BC}}{\sin 143.13}$$

$$\frac{T_{BC}}{\sin 160} = \frac{mg}{\sin 140}$$

$$mg = \frac{T_{BC}}{\sin 160}\times\sin 140 = \frac{29.64}{\sin 160}\times\sin 140$$

$$= 55.7\text{N}$$

$$\text{Mass of Q, m} = \frac{55.7}{9.81} = 5.68\text{kg}$$

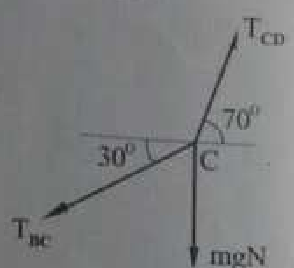


Fig. 1.26

1.8. Principle of superposition

It states that if a number of forces simultaneously act on a body then each one of the forces will produce the same effect, when this force acts alone in the body.

Consider a beam AB subjected to forces F_1 and F_2 as shown in Fig. 1.27. Let R_A and R_B be the reaction at A and B due to combined effect of F_1 and F_2. Let R_{A1} and R_{B1} be the reactions at A and B when the force F_1 alone acts. Let R_{A2} and R_{B2} be the reactions at A and B when the force F_2 alone acts. Then according to principle of superposition, $R_A = R_{A1} + R_{A2}$ and $R_B = R_{B1} + R_{B2}$.

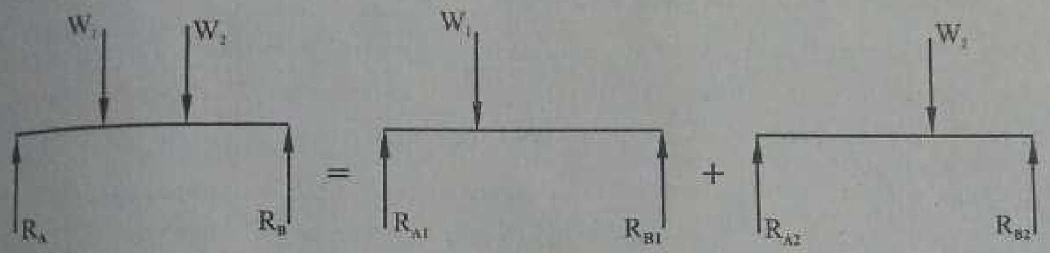


Fig. 1.27

1.9. Law of action and reaction

Newton's third law states that to every action there is an equal and opposite reaction. This law is referred to as law of action and reaction. A body is not always free to move in all directions. The restriction to the free motion of a body is called constraint. Consider a body of weight W resting on a horizontal surface as shown in Fig. 1.28. The body is free to move in all the directions except vertically downwards. The horizontal surface prevents the body from moving verticaly downward. The body presses the surface downward with a force equal to the weight of the body. This downward force is the action of body on the horizontal surface. Refer Fig. 1.29. The horizontal surface resists this action of body and develops an equal force in the upward direction. This force applied by the surface on the body is called reaction. Refer Fig. 1.30. The force exerted by the body on the support is called action and the force exerted by the support on the body is called reaction. For equilibrium of the body, $R - W = 0$. $R = W$. The action and reaction are always equal in magnitude and opposite in direction.

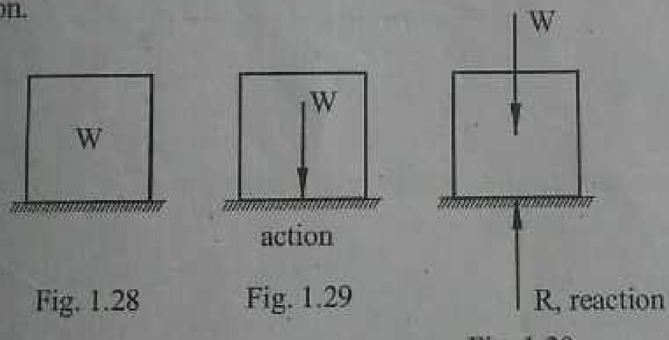


Fig. 1.28 Fig. 1.29 Fig. 1.30

1.10. Concurrent coplanar forces.

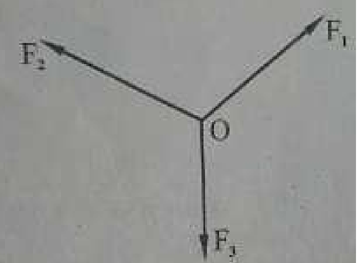


Fig. 1.31 Concurrent forces

A force is characterised by its magnitude, point of application and direction. When several forces act on a body, they are called a force system or a system of forces. When all these forces lie in the same plane, it is called a coplanar system of forces. The coplanar force systems may be collinear, concurrent or non-concurrent. The forces whose lines of action meet at a point are called concurrent forces. The three forces, F_1, F_2 and F_3 as shown in Fig 1.31 are concurrent forces.

1.11. Composition and resolution of forces.

The replacement of several forces by a single force is called composition. The replacement of a single force by several forces is called resolution. The several forces by which a single force can be replaced are called components of the single force.

Consider a force F acting at a point O as shown in Fig 1.32. It is required to replace this force F by two forces F_x and F_y acting along OX and OY directions. These forces F_x and F_y are called the components of F along OX and OY directions respectively. Let θ and φ be the inclination of OX and OY axes with the given force F which is represented by the vector OA.

From A draw lines AC and AB parallel to OX and OY axes. Applying sine rule to the triangle OAB,

$$\frac{OB}{\sin\phi} = \frac{OA}{\sin\ (180 - (\theta + \phi))} = \frac{AB}{\sin\theta}$$

$$\frac{F_x}{\sin\phi} = \frac{F}{\sin\ (\theta + \phi)} = \frac{F_y}{\sin\theta}$$

$$F_x = \frac{F\sin\phi}{\sin\ (\theta + \phi)}$$

$$F_y = \frac{F\sin\ \theta}{\sin\ (\theta + \phi)}$$

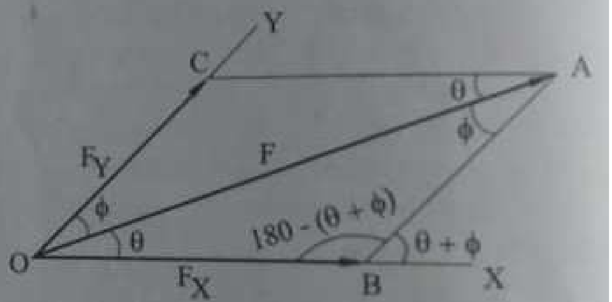


Fig. 1.32

When $\theta + \phi = 90^0$, the components are called rectangular components,

$$F_x = \frac{F\sin\phi}{\sin\ (\theta + \phi)}$$

$$= \frac{F\sin\phi}{\sin 90} = F\sin\ (90 - \theta)$$

$$= F\cos\theta$$

$$F_y = \frac{F\sin\theta}{\sin\ (\theta + \phi)}$$

$$F_y = \frac{F\sin\theta}{\sin 90} = F\sin\theta = F\sin(90 - \phi) = F\cos\phi$$

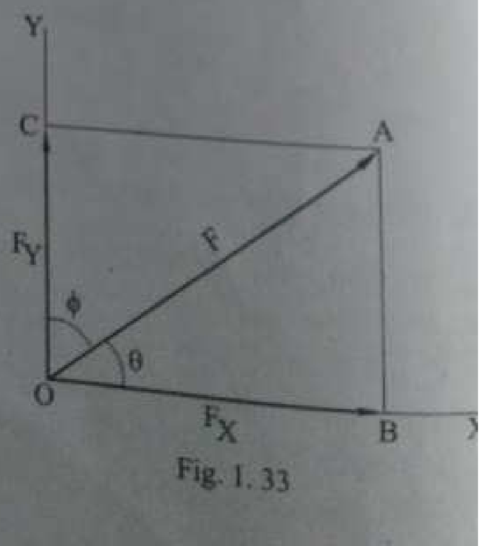


Fig. 1. 33

The rectangular component of a force along a given direction is the product of the magnitude of the force and cosine of the angle between the force and the given direction.

To calculate the rectangular component of a force,

1. Identify the force whose component is required.
2. Identify the direction along which the component is required.
3. Calculate the angle between the force and the direction, if not given.
4. Multiply the force by the cosine of the angle between the force and the direction.

Refer Fig. 1.34. The angle between force F and X axis is θ. Therefore rectangular component of the force F along the X axis, $F_x = F\cos\theta$

The angle between the force F and Y axis is $(90 - \theta)$. Therefore the rectangular component of force F along Y axis,

$$F_y = F\cos(90 - \theta)$$
$$= F\sin\theta.$$

Refer Fig.1.35. The angle between force F and Y axis is θ.

$$\therefore F_y = F\cos\theta \text{ and}$$
$$F_x = F \times \cos(90 - \theta)$$
$$= F\sin\theta.$$

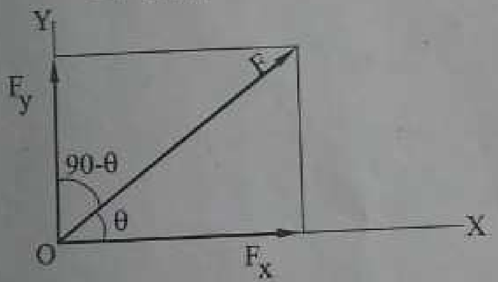


Fig. 1.34

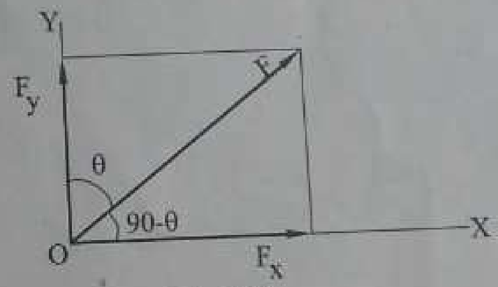


Fig. 1.35

Direction of component of a force.

The direction of component of a force is the direction of the force itself, when the angl between the force and the direction is assumed to be zero.

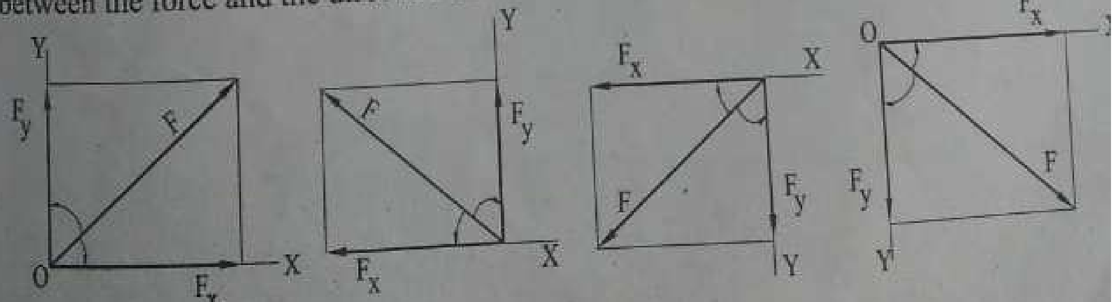
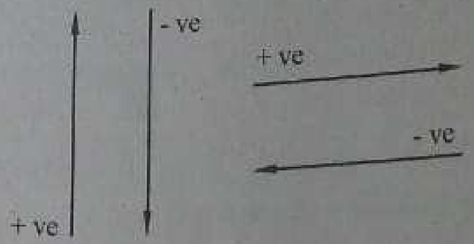


Fig. 1.36

Sign convention for forces.

Forces acting in opposite directions should be given opposite sign. Generally upward force is taken as positive and downward force negative. Force towards right is taken as positive and force acting towards left is taken as negative.



1.12. Method of projections.

Resultant of a number of coplanar concurrent forces can be obtained analytically by resolving the forces along any two mutually perpendicular directions. The force equal in magnitude and opposite in direction of the resultant is called equilibriant. It is the single force required to keep the force system in equilibrium. Th algeibric sum of projetions of a number of coplanar concurrent forces along any direction will be equal to the projection of their resultant along the same direction . This statement is the extention of theorem of resolved parts to any number of forces. The theorem of resolved parts states that projection of the resultant of two forces on any axis is equal to the algebric sum of the projections of the two forces along the same axis.

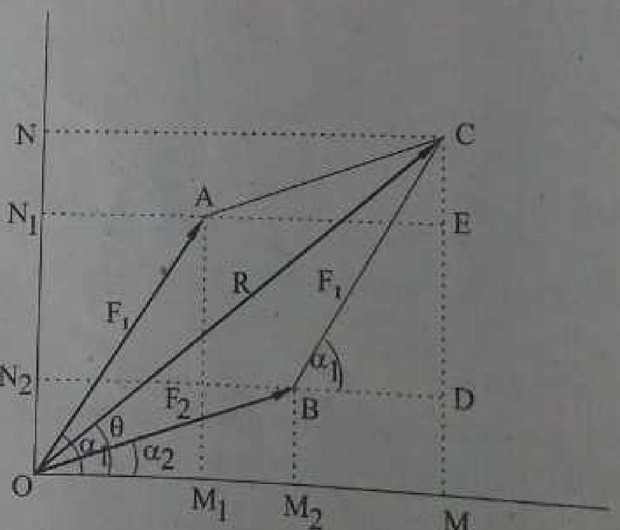


Fig. 1.37

Consider two forces F_1 and F_2 acting at O. Let R be the resultant of F_1 and F_2. It is requir to prove that,

$$F_1 \cos \alpha_1 + F_2 \cos \alpha_2 = R \cos \theta$$

$$\text{and } F_1 \sin \alpha_1 + F_2 \sin \alpha_2 = R \sin \theta$$

$$OM_2 + M_2M = OM$$

$$F_2 \cos\alpha_2 + OM_1 = OM$$

$$F_2 \cos\alpha_2 + F_1 \cos\alpha_1 = OM = R\cos\theta$$

$F_1 \cos\alpha_1 + F_2 \cos\alpha_2 = R\cos\theta$. ie., sum of components of F_1 and F_2 along OX = component of resultant of F_1 and F_2 along OX.

$$ON_1 + N_1N = ON$$

$$F_1 \sin\alpha_1 + AC\sin\alpha_2 = R\sin\theta$$

$$F_1 \sin\alpha_1 + OB\sin\alpha_2 = R\sin\theta$$

$F_1 \sin\alpha_1 + F_2 \sin\alpha_2 = R\sin\theta$ ie., the sum of components of F_1 and F_2 along OY = component of resultant of F_1 and F_2 along OY.

1.13. Resultant of coplanar concurrent forces

Resultant of coplanar concurrent forces can be obtained by applying method of projections.

Consider four forces F_1, F_2, F_3 and F_4 as shown in Fig. 1.38. Sum of components of these forces along the X axis is = the component of their resultant along the X axis. Let R be the resultant of these forces and θ be the inclination of the resultant R with the X axis. According to principle of projections,

$$F_1 \cos\alpha_1 + F_2 \cos\alpha_2 + F_3 \cos\alpha_3 + F_4 \cos\alpha_4 = R\cos\theta$$

$$\Sigma F_x = R.\cos\theta \text{ ------ (1)}$$

$$F_1 \sin\alpha_1 + F_2 \sin\alpha_2 + F_3 \sin\alpha_3 + F_4 \sin\alpha_4 = R\sin\theta$$

$$\Sigma F_y = R.\sin\theta \text{ ----- (2)}$$

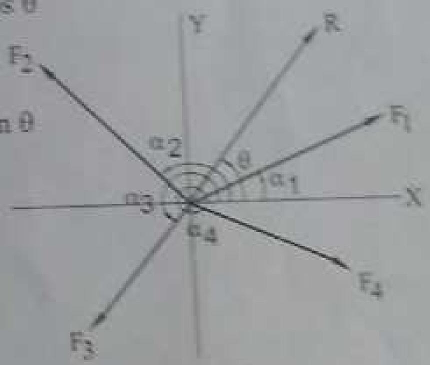


Fig. 1.38

Squaring and adding eqns. (1) and (2),

$$\Sigma F_x^2 + \Sigma F_y^2 = R^2\cos^2\theta + R^2\sin^2\theta$$

$$= R^2(\cos^2\theta + \sin^2\theta)$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

Inclination of resultant with horizontal is given by,

$$\tan\theta = \frac{|\Sigma F_y|}{|\Sigma F_x|}$$

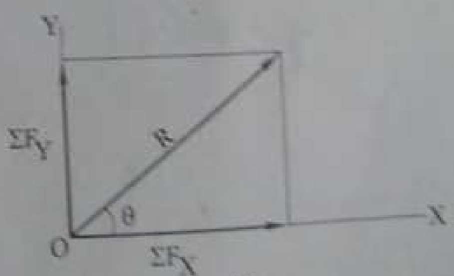


Fig. 1.39

$$\theta = \tan^{-1}\frac{|\Sigma F_y|}{|\Sigma F_x|}$$

When ΣF_x is – ve and ΣF_y is + ve, R will be in the second quadrant.

Inclination of R with horizontal,

$$\theta = \tan^{-1}\frac{|\Sigma F_y|}{|\Sigma F_x|}$$

$$\theta_R = 180 - \theta$$

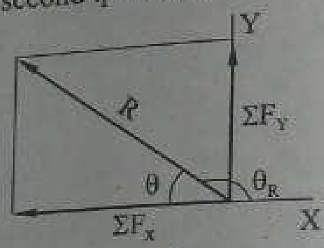


Fig. 1.40

When ΣF_x and ΣF_y are – ve, the resultant R will be in the third quadrant.

Inclination of R with horizontal,

$$\theta = \tan^{-1}\frac{|\Sigma F_y|}{|\Sigma F_x|}$$

Inclination of resultant, $\theta_R = 180 + \theta$

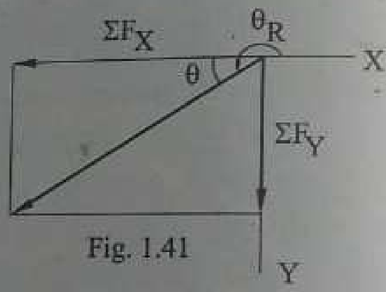


Fig. 1.41

When ΣF_x is +ve and ΣF_y is – ve, the resultant R will be in the fourth quadrant.

Inclination of R with horizontal,

$$\theta = \tan^{-1}\frac{|\Sigma F_y|}{|\Sigma F_x|}$$

$$\theta_R = 360 - \theta$$

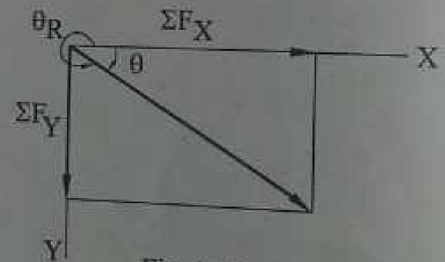


Fig. 1.42

Example 1.9

Find the resultant of the force system shown in Fig 1.43.

Solution.

Resolving the forces along X axis,

$$\Sigma F_x = 300 \sin 30 - 300 \cos 30 - 200 \sin 45$$
$$= -251.23 \text{ N}$$

Resolving the forces along Y- axis,

$$\Sigma F_y = 300 \cos 30 + 300 \sin 30 - 200 \sin 45$$

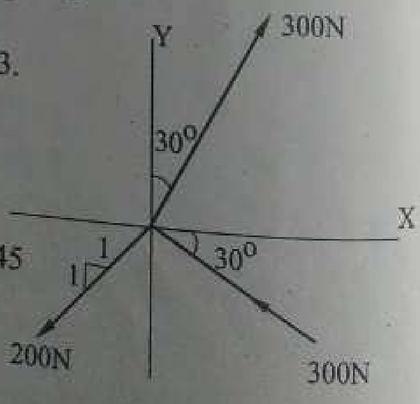


Fig. 1.43

$= 268.39$ N.

Resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(-251.23)^2 + (268.39)^2}$

$R = 367.63$ N

Inclination or resultant with horizontal ,

$\tan\theta = \left|\dfrac{\Sigma F_y}{\Sigma F_x}\right|$

$= \dfrac{268.39}{251.23}$

$= 1.068$

$\theta = \tan^{-1}(1.068)$

$= 46.89^0$

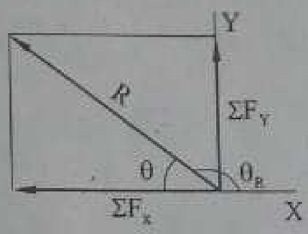


Fig. 1.44

Since the resultant is in the second quadrant, the inclination of the resultant,

$\theta_R = 180 - \theta = 180 - 46.89 = 133.11^0$

Example 1.10

Three wires exert tensions on a small ring as indicated in the Fig. 1.45. Determine the force in a single wire which will replace the three wires.

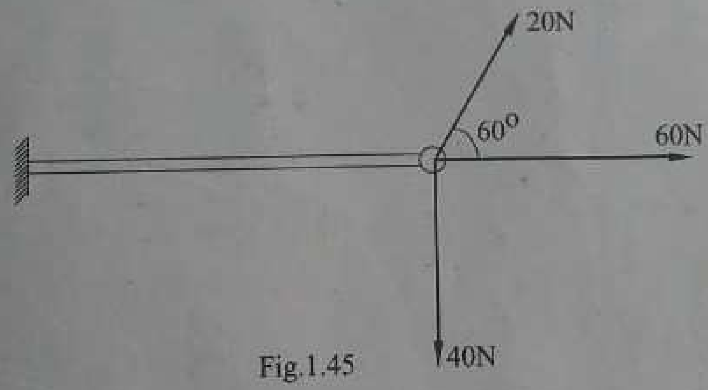


Fig.1.45

Solution.

Resolving the forces along X axis,

$\Sigma F_x = 60 + 20\cos 60 = 70$ N

Resolving the forces along Y axis,

$\Sigma F_y = 20\sin 60 - 40 = -22.68$ N

Resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(70)^2 + (-22.68)^2}$

$= 73.58$ N

Inclination of resultant with horizontal,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$

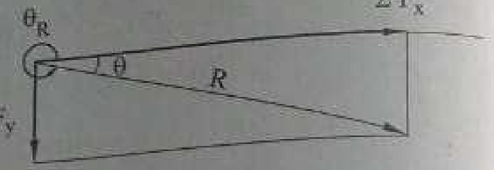


Fig. 1.46

$$= \tan^{-1}\frac{22.68}{70} = \tan^{-1}0.324$$

$= 17.95^0$

Inclination of resultant, $\theta_R = 360 - \theta$

$= 360 - 17.95$

$= 342.05^0$

Example 1.11

Find the resultant of the forces shown in Fig 1.47.

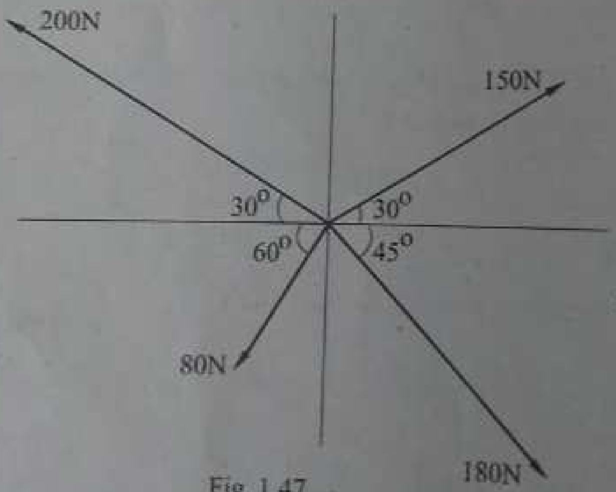


Fig. 1.47

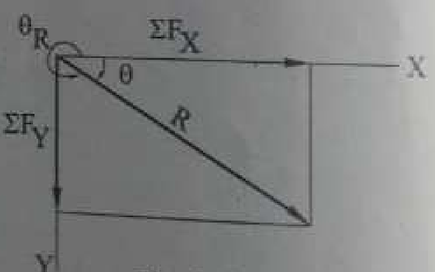


Fig. 1.48

Solution

Resolving the forces along X axis,

$$\Sigma F_x = 150 \cos 30 + 180 \cos 45 - 200 \cos 30 - 80 \cos 60$$

$$= 43.98 \text{ N}$$

Resolving the forces along the Y axis

$$\Sigma F_y = 150 \sin 30 + 200 \sin 30 - 80 \sin 60 - 180 \sin 45$$
$$= -21.56 \text{ N}$$

$$\text{Resultant } R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$= \sqrt{(43.98)^2 + (-21.56)^2}$$
$$= 48.98 \text{ N}$$

Inclination of resultant with horizontal,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$
$$= \tan^{-1}\frac{21.56}{43.98}$$
$$= 26.12^0$$

Since the resultant is in the fourth quadrant, the inclination of resultant,

$$\theta_R = 360 - \theta$$
$$= 360 - 26.12$$
$$= 333.88^0$$

Example 1.12 [KTU June 2016]

Determine the magnitude and direction of the resultant of the forces acting on the ring as shown in Fig. 1.49.

Solution

Resolving the forces horizontally,

$$\Sigma F_H = 20 + 25\cos 70 - 70\cos 70 - 30\cos 20 - 40\cos 40$$
$$= -54.22\text{kN} = 54.22\text{kN towards left}$$

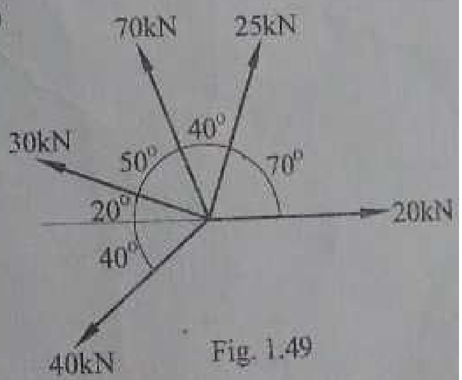


Fig. 1.49

Resolving the forces vertically,

$$\Sigma F_V = 0 + 25\sin 70 + 70\sin 70 + 30\sin 20 - 40\sin 40$$
$$= 73.82\text{kN}$$

$$\text{Resultant, } R = \sqrt{\Sigma F_H^{\ 2} + \Sigma F_V^{\ 2}}$$
$$= \sqrt{54.22^2 + 73.82^2} = 91.59\text{kN}$$

Inclination of resultant with horizontal,

$$\tan\theta = \frac{\Sigma F_V}{\Sigma F_H} = \frac{73.82}{54.22} = 1.361$$

$$\theta = 53.7^\circ$$

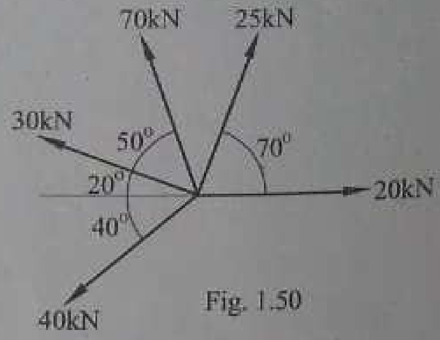


Fig. 1.50

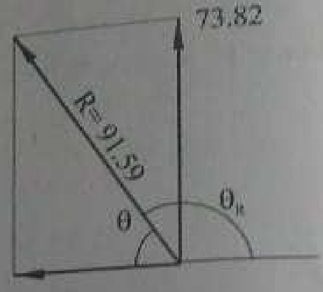


Fig. 1.51

Since the resultant is in the second quadrant,

$$\theta_R = 180 - \theta$$

$$= 180 - 53.7 = 126.3^\circ$$

Example 1.13 [KTU Aug 2016]

Forces of 15N, 20N, 25N, 35N and 45N act at an angular point of a regular hexagon towards the other angular points as shown in Fig.1.52. Calculate the magnitude and direction of the resultant force.

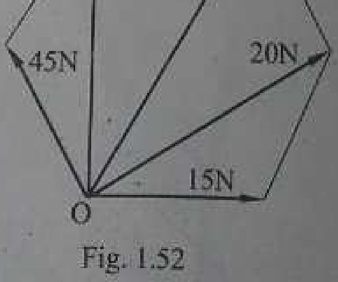


Fig. 1.52

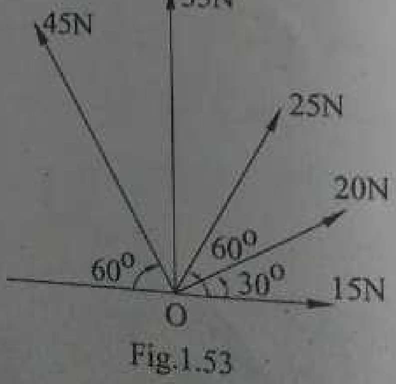


Fig.1.53

Solution.

Resolving the forces along X axis,

$$\sum F_x = 15 + 20\cos 30 + 25\cos 60 + 0 - 45\cos 60 = 22.32\ N$$

Resolving the forces along y axis,

$$\Sigma F_y = 0 + 20 \sin 30 + 25 \sin 60 + 35 + 45 \sin 60 = 105.62 \text{ N}$$

$$\text{Resultant, } R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$= \sqrt{22.32^2 + 105.62^2}$$

$$= 107.95 \text{ N}$$

Inclination of resultant with horizontal,

$$\theta = \tan^{-1}\left|\frac{\Sigma F_y}{\Sigma F_x}\right|$$

$$= \tan^{-1}\frac{105.62}{22.32}$$

$$= 78.07^0$$

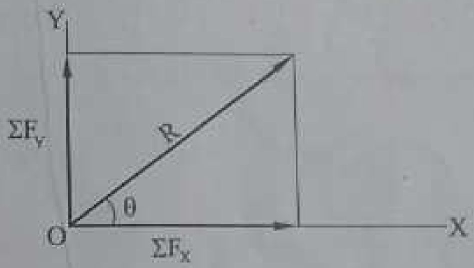


Fig. 1.54

Inclination of resultant, $\theta_R = \theta = 78.07^0$

Example 1.14 [KTU July 2016]

Concurrent forces of 1,3,5,7,9,11 are applied to the center of a regular hexagon acting towards its vertices as shown in Fig. 1.55. Determine the magnitude and direction of the resultant.

Solution

In a regular hexagon the angle substended by each side at the center is 60°.

Resolving the forces horizontally,

$$\Sigma F_H = 5 + 3\cos 60 - 1\cos 60 - 11 - 9\cos 60 + 7\cos 60$$

$$= -6N = 6N \text{ towards left}$$

Resolving the forces vertically,

$$\Sigma F_V = 0 + 3\sin 60 + 1\sin 60 + 0 - 9\sin 60 - 7\sin 60$$

$$= -10.39N$$

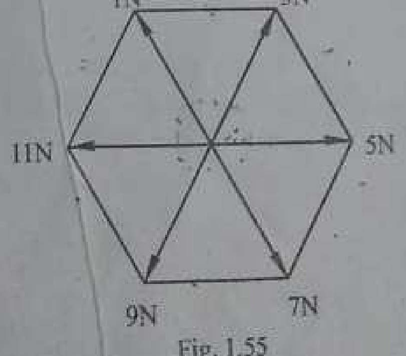


Fig. 1.55

$$\text{Resultant, } R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2}$$

$$= \sqrt{-6^2 + -10.39^2} = 12N$$

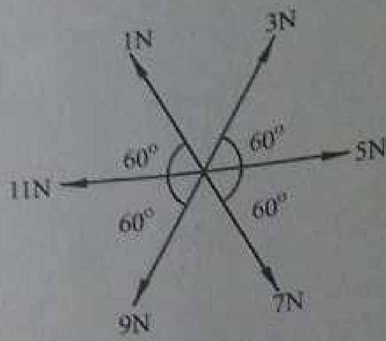


Fig. 1.56

Inclination of resultant with horizontal,

$$\tan\theta = \frac{\Sigma F_V}{\Sigma F_H} = \frac{10.39}{6} = 1.732$$

$$\theta = 60^\circ$$

Since the resultant is in the third quadrant,

$$\theta_R = 180 + \theta$$

$$= 180 + 60 = 240^\circ$$

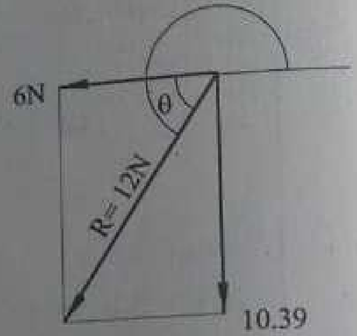


Fig. 1.57

Example 1.15.

The direction of the 300N forces may vary but the angle between the two forces is always 40°. Determine the value of α for which the resultant of the forces acting at A is directed along the plane b-b as shown in Fig.1.58

Solution.

For the resultant force to be along b-b, the sum of components of all the forces perpendicular to the plane b-b should be zero.

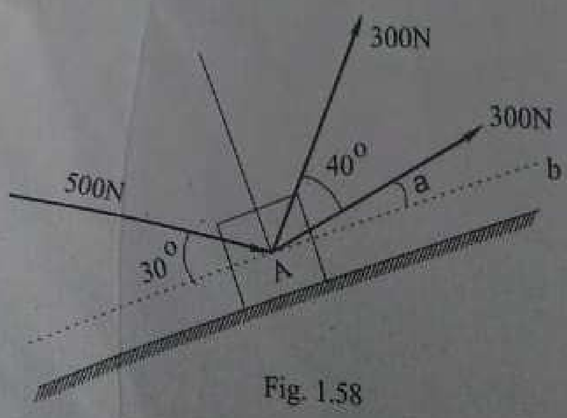


Fig. 1.58

ie., $300 \sin \alpha + 300 \sin (40 + \alpha) - 500 \sin 30 = 0$

$$\sin \alpha + \sin (\alpha + 40) = 0.833$$

$$2\sin \frac{(\alpha + \alpha + 40)}{2} \cos \frac{(\alpha - \alpha - 40)}{2} = 0.833$$

$$2 \sin (\alpha + 20). \cos (-20) = 0.833$$

$$\sin (\alpha + 20) = \frac{0.833}{2 \times \cos 20} = 0.443$$

$$\alpha + 20 = \sin^{-1} (0.443) = 26.31^0$$

$$\alpha = 26.31 - 20 = 6.31^0$$

$$\alpha = 6.31^0$$

1.14. Equilibrium equations.

A number of forces acting on a particle is said to be in equilibrium when their resultant force is zero. If the resultant force is not equal to zero, then the particle can be brought to rest by applying a force equal and opposite to the resultant force. Such a force is called equilibriant. Resultant and equilibriant are equal in magnitude and opposite in direction. Resultant of a number of coplanar concurrent forces is given by

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$, where ΣF_x and ΣF_y are the sum of components of all the forces along two mutually perpendicular X and Y directions. For R to be zero, both ΣF_x and ΣF_y must be zero. Therefore the equations of equilibrium are,

$$1) \Sigma F_x = 0 \quad \text{and} \quad 2) \Sigma F_y = 0$$

Example 1.16.

A rope 9m long is connected at A and B, two points on the same level, 8m apart. A load of 300N is suspended from a point C on the rope, 3m from A. What load connected to a point D, on the rope, 2m from B is necessary to keep portion CD parallel to AB.

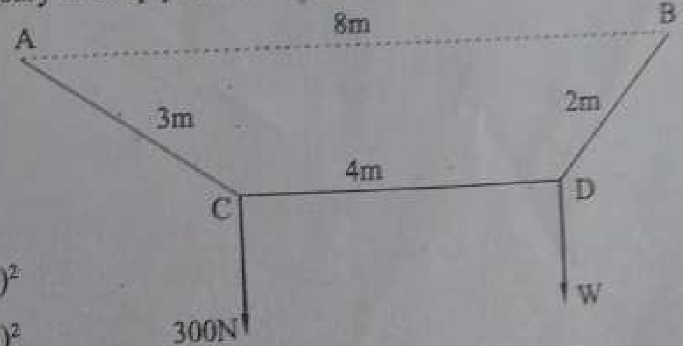


Fig. 1.59

Solution.

From the triangle ACE,

$$y^2 = 3^2 - x^2$$

From the triangle BDF,

$$y^2 = 2^2 - (4 - x)^2$$

$$3^2 - x^2 = 2^2 - (4 - x)^2$$

$$9 - x^2 = 4 - 16 - x^2 + 8 x$$

$8x = 21$

$x = 2.625$ m

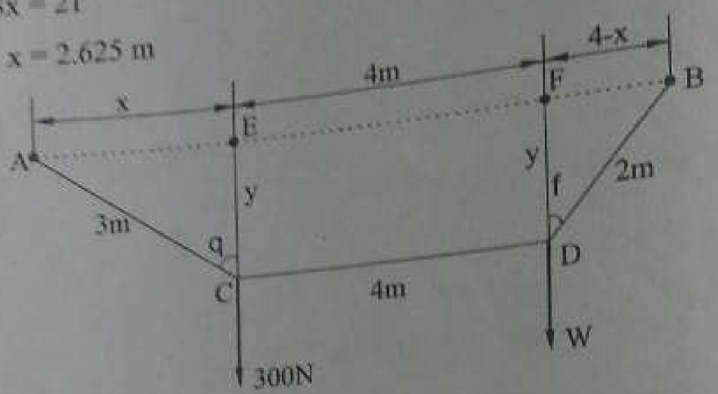


Fig. 1.60

$$\sin\theta = \frac{x}{3} = \frac{2.625}{3} = 0.875$$

$$\theta = 61.04^0$$

$$\sin\phi = \frac{4-x}{2} = \frac{4-2.625}{2} = 0.6875$$

$$\phi = 43.43^0$$

Consider the equilibrium of point C.

Resolving the forces vertically,

For $\Sigma F_V = 0$

$$T_{AC}\cos\theta - 300 = 0$$

$$T_{AC} = \frac{300}{\cos 61.04} = 619.58 \text{ N}$$

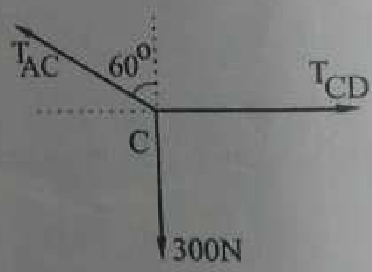


Fig. 1.61

Resolving the forces horizontally,

or $\Sigma F_H = 0$

$$T_{CD} - T_{AC}\sin\theta = 0$$

$$T_{CD} = T_{AC}\sin\theta = 619.58 \sin 61.04$$

$$= 542.11 \text{ N.}$$

Considering the equilibrium of point D

For $\Sigma F_H = 0$

$$T_{DB}\sin\phi - T_{CD} = 0$$

$$T_{DB} = \frac{T_{CD}}{\sin \phi} = \frac{542.11}{\sin 43.43^0}$$

$$= 788.56 \text{ N}$$

For $\sum F_V = 0$

$T_{DB} \cos \phi - W = 0$

$W = T_{DB} \cos \phi = 788.56 \cos 43.43$

$W = 572.66$ N

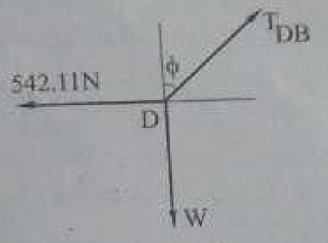


Fig. 1.62

Example 1.17

A roller of radius 300 mm and weight 1000N is to be pulled over a rectangular block of height 150 mm as shown in Fig 1.63. Determine (i) the horizontal force required to be applied through the centre O and (ii) the required horizontal force when it is applied through the top end of vertical diameter.

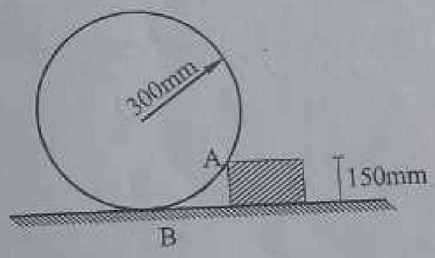


Fig. 1.63

Solution.

Case (i)

Horizontal force is applied through the centre.

When the roller is just turned about A, the contact at B breaks and hence there is no reaction at B. Let P be the applied force and R be the reaction at the contact point A.

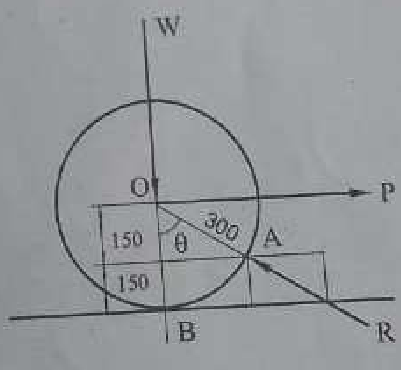


Fig. 1.64

$$OB = OA \cos \theta + 150$$

$$300 = 300 \cos \theta + 150$$

$$\theta = 60^0$$

Resolving the forces vertically,

For $\sum F_V = 0$

$R \cos \theta - 1000 = 0$

$R \cos 60 = 1000$

$$R = \frac{1000}{\cos 60} = 2000 \text{ N}$$

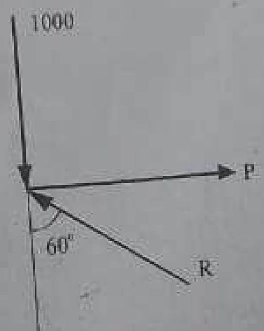


Fig. 1.65

Resolving the forces horizontally,

For $\sum F_H = 0$

$P - R \sin\theta = 0$

$P = R \sin\theta = 2000 \sin 60$

$= 1732.05$ N

Case (ii) When the force P is applied through the top end of the diameter.

The line of action of R should intersect at C, where the line of action of other two forces intersect. Triangle OAC is an isosceles triangle with $\angle AOC = 120^0$

$$\therefore \alpha = \frac{180 - 120}{2} = 30^0$$

Resolving the forces vertically,

For $\sum F_V = 0$,

$R \cos 30 - W = 0$

$$R = \frac{W}{\cos 30} = \frac{1000}{\cos 30} = 1154.7 \text{ N}$$

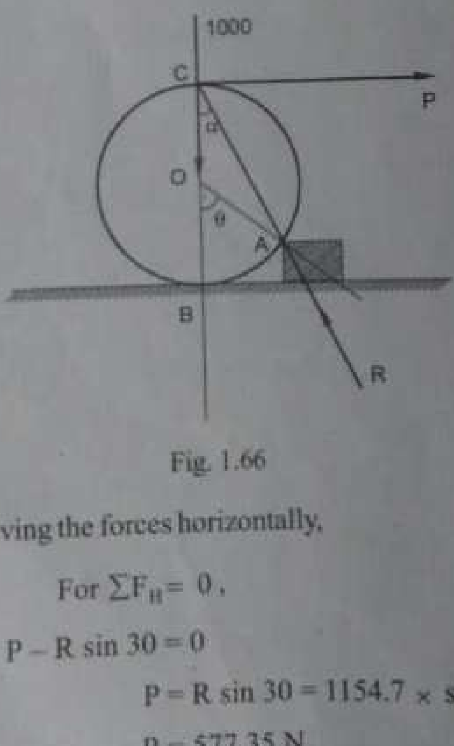


Fig. 1.66

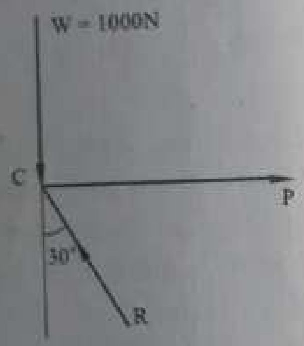


Fig. 1.67

Resolving the forces horizontally,

For $\sum F_H = 0$,

$P - R \sin 30 = 0$

$P = R \sin 30 = 1154.7 \times \sin 30$

$P = 577.35$ N.

1.12. Free-body diagram.

The problems of equilibrium are derived from the physical situation of a system. A sketch showing the physical situation of a system consisting of several bodies is called a space diagram. For a system consisting of several bodies to be in equilibrium each body of the system, considered separately must be in equilibrium. To investigate the equilibrium of a certain body in a system, it should be assumed that the body is isolated from all other supporting bodies. After isolating the body, all the applied forces on the body, self-weight of the body and the reactions at the points of contact with other supporting bodies are shown by directed lines. This sketch in which the body is completely isolated from its supports and in which all the forces acting on it are shown is called a free- body diagram. The free-body diagram makes it easy to apply the laws of equilibrium to the set of forces acting on the body and is of considerable help in solving complicated problems. Fig 1.69 shows the free-body diagrams of two spheres kept inside a cylindrical vessel.

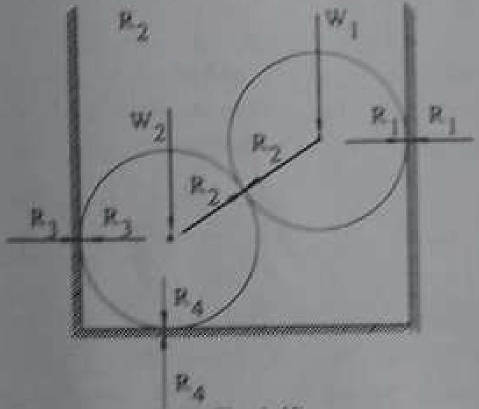


Fig. 1.68

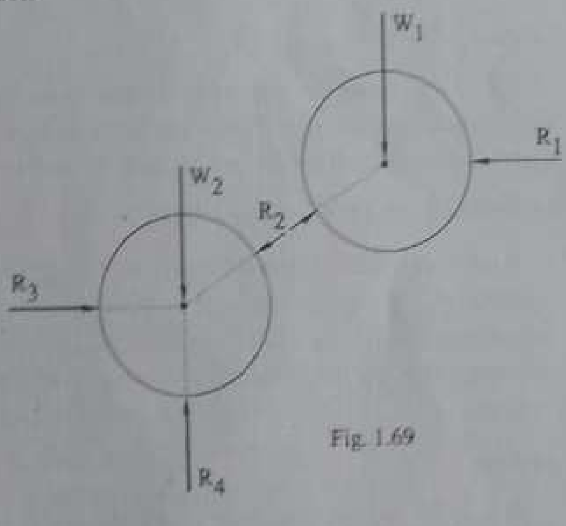


Fig. 1.69

Example 1.18 [KTU June 2016]

Three smooth identical spheres A, B and C are placed in a rectangular channel as shown in Fig. 1.70. Draw the free body diagram of each spheres.

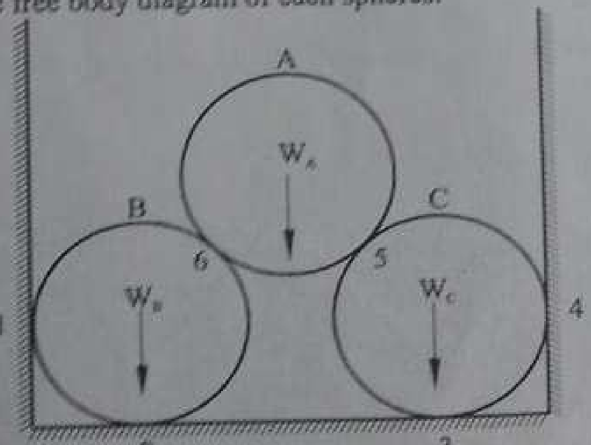


Fig. 1.70

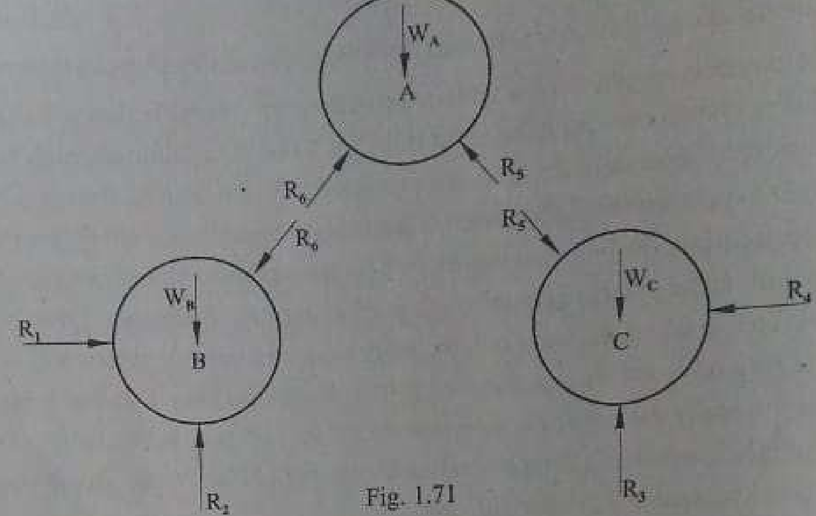


Fig. 1.71

Solution

Fig. 1.71 shows the free body diagrams of the spheres. R_1, R_2, R_3 and R_4 are perpendicular to the contact surfaces and reaction R_5 is directed along the line joining the centres of spheres A and C. Reaction R_6 is directed along the line joining the centres of spheres A and B.

Example 1.19 [KTU May 2019]

A solid cylinder 30mm diameter and weighing 30N is placed in a triangular channel as shown in Fig. 1.72. Neglecting the friction between the contact surfaces, calculate the normal reaction on the sides of the channel.

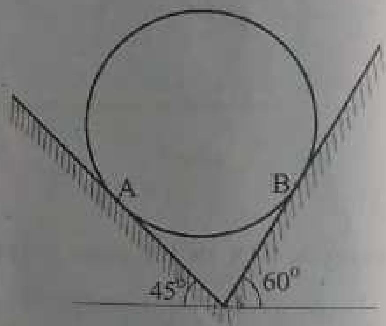


Fig. 1.72

Solution

Fig. 1.73 shows the free body diagram of the cylinder. R_A and R_B are perpendicular to the contact surface at A and B.

Applying Lami's theorem,

$$\frac{30}{\sin 105} = \frac{R_A}{\sin(180-60)} = \frac{R_B}{\sin(180-45)}$$

$$R_A = \frac{30}{\sin 105} \times \sin(180-60) = 26.89\text{N}$$

$$R_B = \frac{30}{\sin 105} \times \sin(180-45) = 21.96\text{N}$$

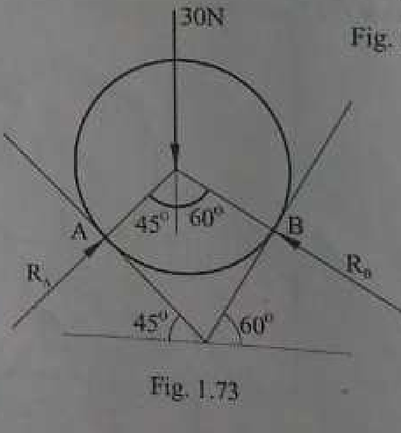


Fig. 1.73

Example 1.20.

A steel ball rests in a groove, the sides of which are smooth. One side of the groove is vertical while the other side is at 40° to the horizontal. If the ball has a weight of 10 N find the reaction on each wall of the groove.

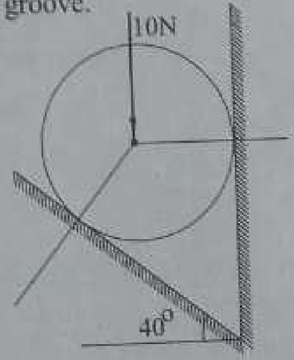


Fig. 1.74

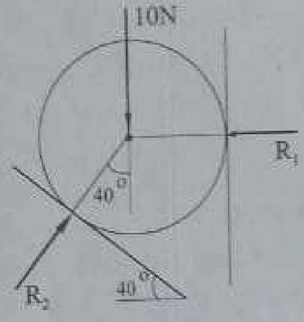


Fig. 1.75

Solution.

For $\Sigma F_V = 0$

$R_2 \cos 40 - 10 = 0$

$$R_2 = \frac{10}{\cos 40} = 13.05 \text{ N}$$

For $\Sigma F_H = 0$

$R_2 \sin 40 - R_1 = 0.$

$R_1 = R_2 \sin 40 = 13.05 \sin 40$

$= 8.39$ N

Reaction on vertical wall = 8.39 N.

Reaction on inclined wall = 13.05 N.

Example 1.21.

A smooth sphere of weight W is supported in contact with a smooth vertical wall by a string fastened to a point in its surface, the other end being attached to a point in the wall. If the length of the string be equal to the radius of the sphere find the inclination of the string to the vertical, the tension in the string and the reaction of the wall.

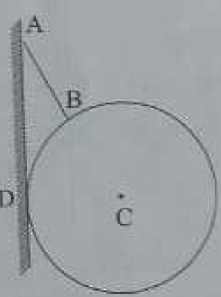


Fig. 1.76

Solution.

Let T be the tension in the string and R be the reaction of the wall.

$$\cos\theta = \frac{r}{2r} = \frac{1}{2}$$

$\therefore \theta = 60^0$

For $\Sigma F_y = 0,$

$T \sin\theta - W = 0$

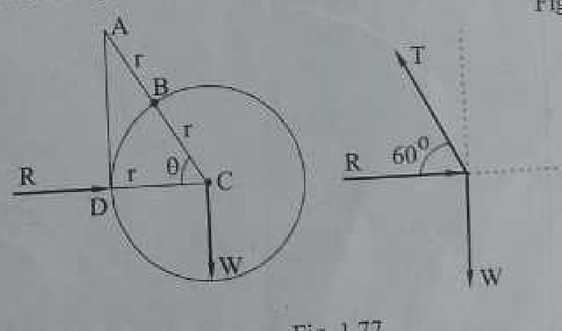


Fig. 1.77

$$T = \frac{W}{\sin 60} = 1.154\ W$$

For $\sum F_x = 0$

$T \cos 60 - R = 0$

$R = T \cos 60$

$= 1.154\ W \times \cos 60$

$R = 0.577\ W$

Example 1.22

A roller of weight W = 1000N rests on a smooth inclined plane and is kept from rolling down by a string AC as shown in Fig.1.78. Using method of projections, find the tension S in the string and the reaction at the point of contact B.

Solution.

Let T be the tension in the string and R be the reaction at B.

Resolving the forces along X direction,

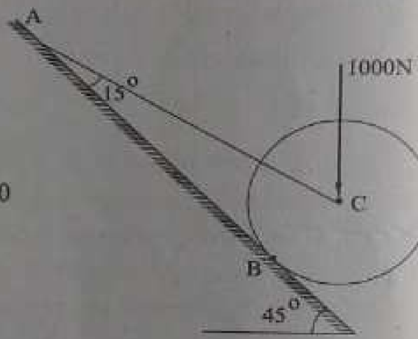


Fig. 1.78

$$\sum F_X = 0$$

$R \cos 45 - S \cos 30 = 0$ ---------(i)

Resolving the forces along Y direction,

For $\sum F_y = 0$

$R \sin 45 + T \sin 30 - 1000 = 0$ ---------(ii)

From eqns (i),

$R \cos 45 = T. \cos 30.$

$R \sin 45 = T \cos 30$

From eqn (ii),

$T \cos 30 + T \sin 30 - 1000 = 0$

$T (\cos 30 + \sin 30) = 1000.$

$$\therefore T = \frac{1000}{(\cos 30 + \sin 30)} = 732.05\ N$$

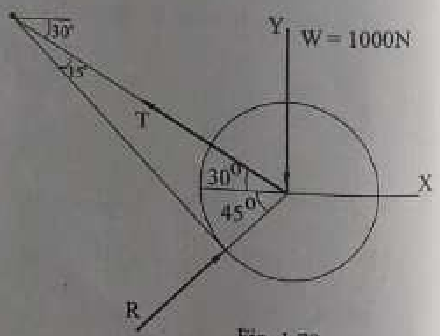


Fig. 1.79

$$\text{Reaction } R = \frac{T \cos 30}{\cos 45} = \frac{732.05 \times \cos 30}{\cos 45}$$

$R = 896.57\ N.$

Example 1.23

Two identical rollers, each of weight Q = 500N are supported by an inclined plane and a vertical wall as shown in Fig 1.80. Assuming smooth surfaces, find the reactions induced at the points of support A,B and C.

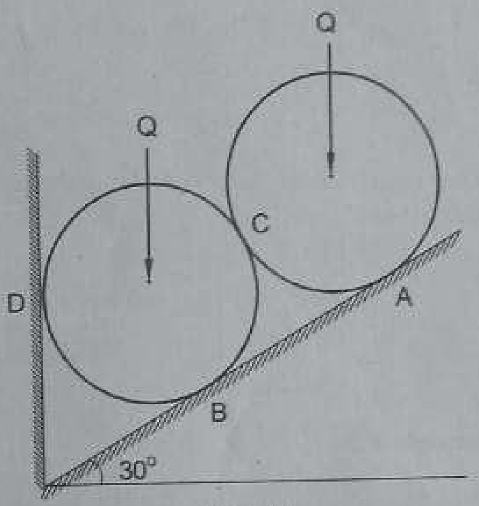


Fig. 1.80

Solution

Consider the upper roller.

Resolving the forces parallel to the inclined plane,

$R_c - Q \sin 30 = 0$

$R_c = Q \sin 30 = 500 \sin 30 = 250$ N

Resolving the forces perpendicular to the inclined plane,

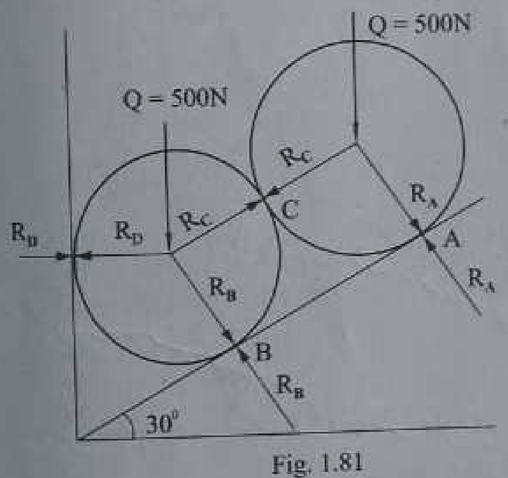


Fig. 1.81

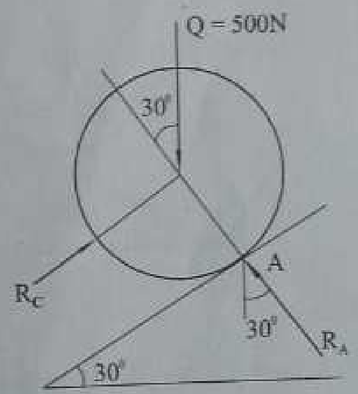


Fig. 1.82

$R_A - 500 \cos 30 = 0,$

$R_A = 500 \cos 30 = 433$ N.

Consider the lower roller, (Refer fig. 1.136)

Resolving the forces along the plane,

$R_D \cos 30 - 250 - 500 \sin 30 = 0$

$$R_D = \frac{500 \sin 30 + 250}{\cos 30}$$

$= 577.35$ N

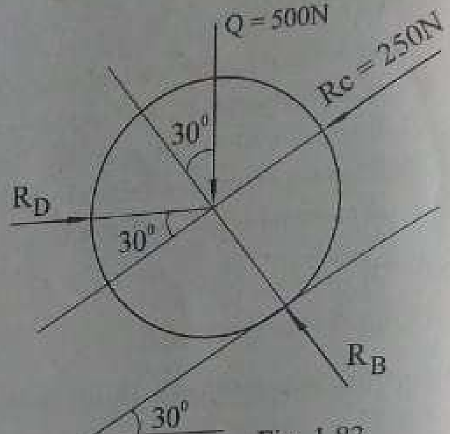


Fig. 1.83

Resolving the forces perpendicular to the inclined plane,

$R_B - 500 \cos 30 - R_D \sin 30 = 0$

$R_B = R_D \sin 30 + 500 \cos 30$

$= 577.35 \sin 30 + 500 \cos 30$

$= 721.69$ N

Example 1.24

Two identical rollers each weighing 400N are supported by an inclined plane inclined at 30° to the horizontal and a wall, at right angles to the inclined plane as shown in the Fig.1.84. Find the reactions at the supports A ,B and C: Neglect friction.

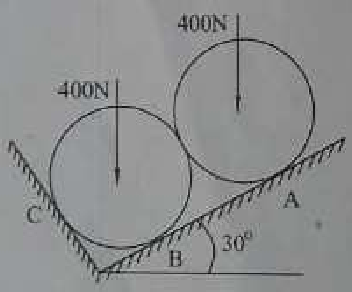


Fig. 1.84

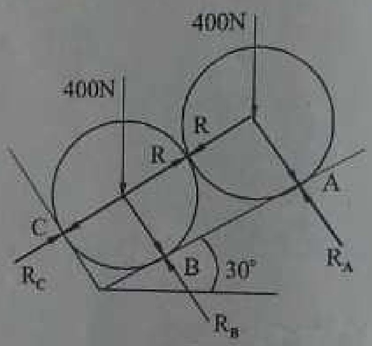


Fig. 1.85

Solution

The freebody diagram of the two rollers is shown in Fig.1.85. Consider the equilibrium of the upper roller. Resolving the forces in the direction of R_A,

$R_A - 400 \cos 30 = 0$

$R_A = 400 \cos 30$

$= 346.41$N.

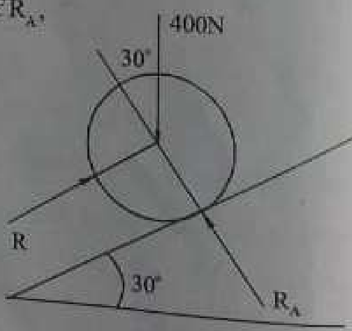


Fig. 1.86

Resolving the forces in the direction of R,

$R - 400 \sin 30 = 0$

$R = 200$ N.

Consider the equilibrium of lower roller. Resolving the forces in the direction of R_B,

$$R_B - 400\cos 30 = 0$$
$$R_B = 400\cos 30$$
$$= 346.41\text{N}.$$

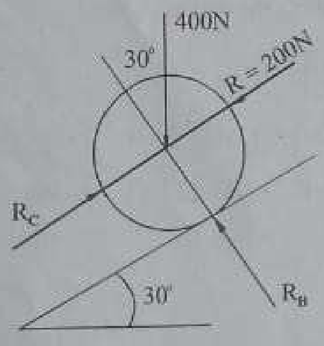


Fig. 1.87

Resolving the forces in the direction of R_C,

$$R_C - 200 - 400\sin 30 = 0$$
$$R_C = 200 + 400\sin 30$$
$$= 400\text{ N}$$

Example 1.25 [KTU Jan 2016]

Two smooth cylinders A and B each of diameter 400mm and weight 200N rest in a horizontal channel having vertical walls and base width of 720mm as shown in Fig 1.88. Find the reaction at P, Q and R.

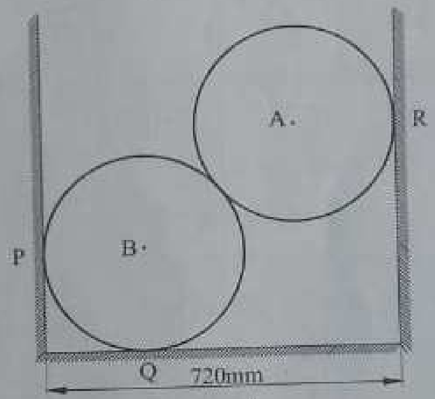


Fig. 1.88

Solution.

$$AB = 400\text{ mm}$$
$$BC = 720 - 400 = 320\text{ mm}$$
$$\cos\theta = \frac{BC}{AB} = \frac{320}{400}$$
$$\theta = 36.87^\circ$$

Consider the equilibrium of upper cylinder

Resolving the forces vertically

For $\Sigma F_V = 0$,

$$R_2\sin\theta - 200 = 0$$
$$R_2 = \frac{200}{\sin 36.87} = 333.33\text{ N}$$

Resolving the forces horizontally,

$$R_2\cos\theta - R_1 = 0$$
$$R_1 = R_2\cos\theta$$
$$= 333.33 \times \cos 36.87$$
$$= 266.67\text{ N}$$

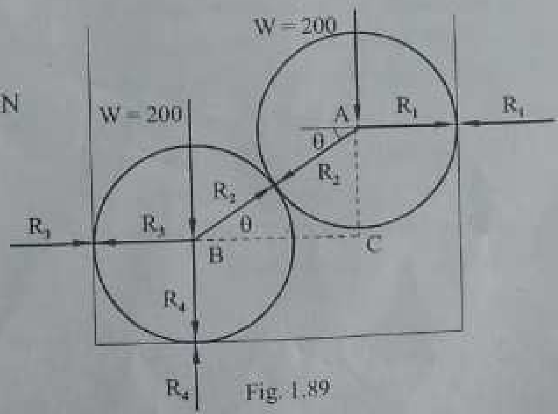


Fig. 1.89

Consider the equilibrium of lower cylinder.

Resolving the forces vertically,

For $\Sigma F_V = 0$

$R_4 - 200 - 333.33 \sin\theta = 0$

$R_4 = 200 + 333.33 \sin 36.87$

$= 400$ N

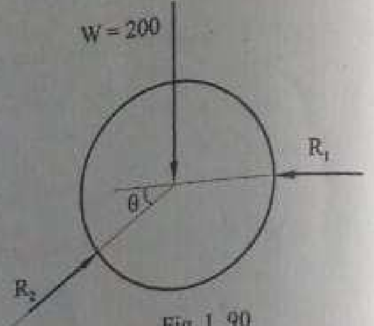


Fig. 1.90

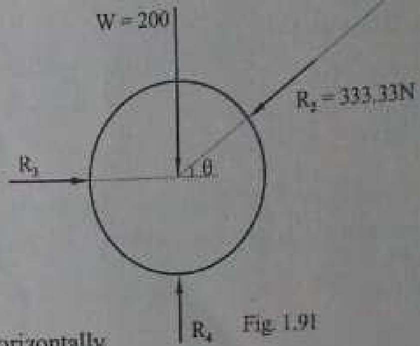


Fig. 1.91

Resolving the forces horizontally,

For $\Sigma F_H = 0$

$R_3 - 333.33 \cos\theta = 0$

$R_3 = 333.33 \cos 36.87$

$= 266.67$ N.

Reaction at P = 266.67 N

Reaction at Q = 400 N

Reaction at R = 266.67 N

Example 1.26

Two smooth circular cylinders each of weight 100N and radius 15 cm are connected at their centres by a string AB of length 40cm. and rest upon a horizontal plane as shown in Fig.1.92. The cylinder above them has a weight 200N and radius of 15cm. Find the force in the string AB and the pressure produced in the floor at the points of contact D and E.

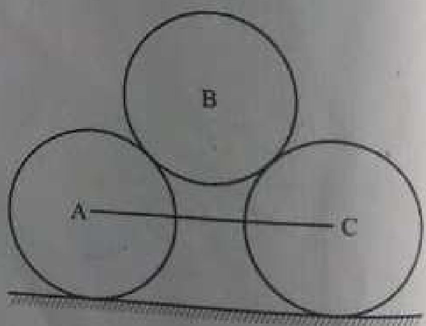


Fig. 1.92

Solution.

Given: $W_A = W_B = 100$ N

$r_A = r_B = 15$ cm

$W_C = 200$ N, $r_C = 15$ cm.

$l = AB = 40$ cm.

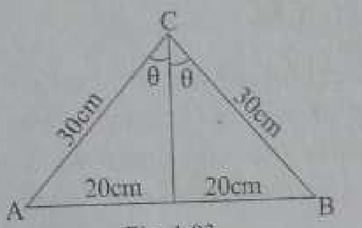


Fig. 1.93

$$\sin\theta = \frac{20}{30}$$

$$\theta = \sin^{-1}\frac{20}{30} = 41.81^0$$

Consider the equilibrium of upper cylinder;

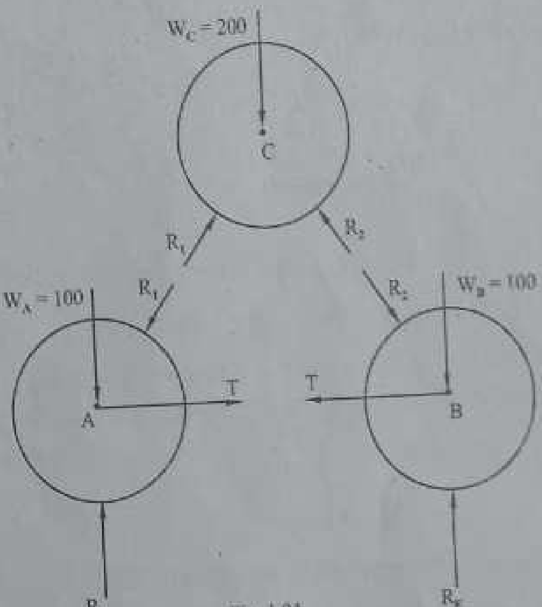


Fig. 1.94

For $\Sigma F_x = 0$

$$R_1 \sin 41.81 - R_2 \sin 41.81 = 0$$

$$R_1 = R_2$$

For $\Sigma F_y = 0$

$$R_1 \cos 41.81 + R_2 \cos 41.81 - 200 = 0$$

$$2 R_1 \cos 41.81 = 200$$

$$R_1 = \frac{200}{2. \cos 41.81} = 134.16N$$

$$R_1 = R_2 = 134.16 \text{ N}$$

Consider the equilibrium of lower cylinder,

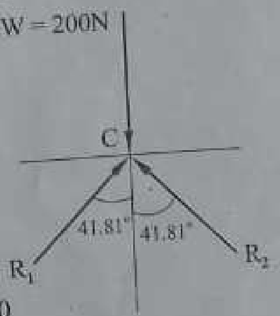


Fig. 1.95

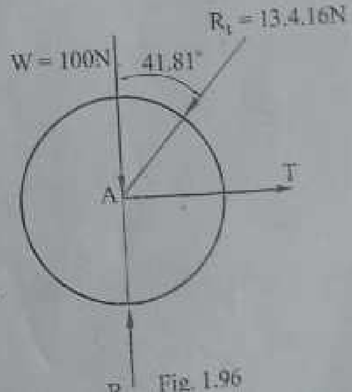


Fig. 1.96

For $\Sigma F_x = 0$

$$T - 134.16 \sin 41.81 = 0$$

$$T = 134.16 \times \sin 41.81$$

$$= 89.44 \text{ N}$$

For $\Sigma F_y = 0$,

$$R_D - 100 - 134.16 \cos 41.81 = 0$$

$$R_D = 100 + 134.16 \cos 41.81$$

$$= 200 \text{ N}$$

Because of symmetrical arrangement of cylinders, the reaction at E will be same as that at D. Therefore, $R_E = R_D = 200$ N

Example 1.27.

Two cylinders A and B rest as shown in Fig 1.97. A has a diameter of 10cm and weight 15N. B has a diameter of 18cm and weight 45N. Determine the pressure at all the points of contact.

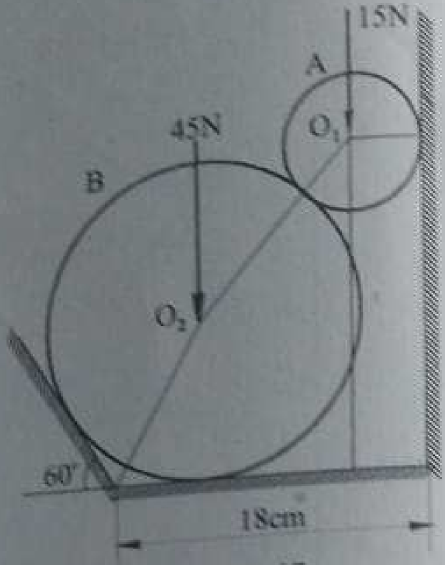


Fig. 1.97

Solution:

Refer Fig. 1.98,

$$\cos\theta = \frac{O_2H}{O_2O_1}$$

$$\frac{ED}{O_2D} = \tan 30$$

$$ED = O_2D \tan 30 = 9 \tan 30 = 5.196 \text{ cm}$$

$$O_2H = DG = EK - GK - ED$$

$$= 18 - 5 - 5.196 = 7.804 \text{ cm.}$$

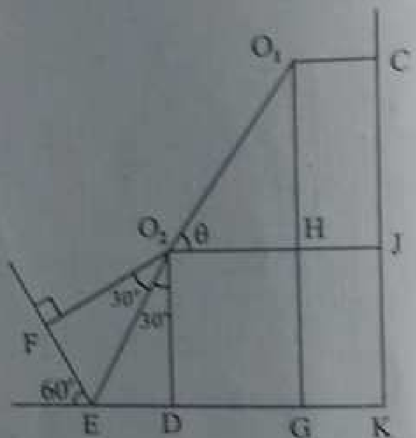


Fig. 1.98

$$\cos\theta = \frac{7.804}{14}$$

$$\theta = 56.12^{\circ}$$

Consider the equilibrium of cylinder A.

Resolving the forces vertically.

For $\Sigma F_V = 0$,

$R \sin\theta - 15 = 0$

$R \sin 56.12 = 15$

$$R = \frac{15}{\sin 56.12} = 18.07 \text{ N}$$

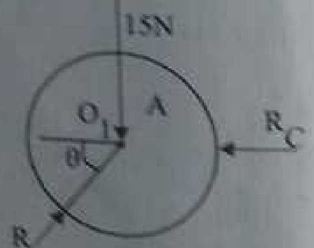


Fig. 1.99

Resolving the forces horizontally,

For $\Sigma F_H = 0$

$R \cos\theta - R_c = 0$

$$R_c = R\cos\theta = 18.07 \cos 56.12 = 10.07$$

$$R_c = 10.07 \text{ N.}$$

Consider the equilibrium of cylinder B.

Resolving the forces horizontally,

For $\Sigma F_H = 0$,

$R_F \sin 60 - 18.07 \cos 56.12 = 0$

$$R_F = \frac{18.07 \cos 56.12}{\sin 60}$$

$$= 11.63 \text{ N}$$

Resolving the forces vertically,

For $\Sigma F_V = 0$,

$R_D + R_F \cos 60 - 18.07 \sin 56.12 - 45 = 0$

$$R_D = 45 + 18.07 \sin 56.12 - 11.63 \cos 60$$

$$R_D = 54.19 \text{ N}$$

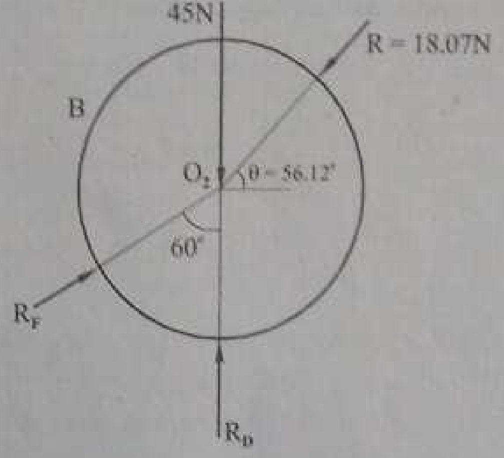


Fig. 1.100

Example 1.28.

Three cylinders are piled in a rectangular ditch as in Fig.1.101. Neglecting friction, determine the reaction between cylinder A and vertical wall

Solution:

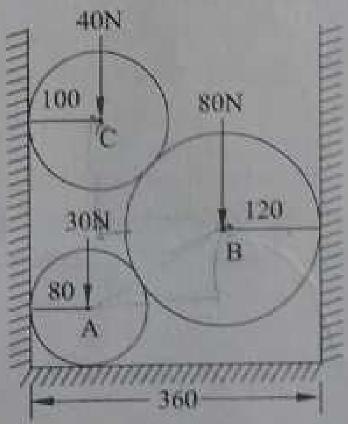


Fig. 1.101

$$\cos\theta_1 = \frac{BE}{BC} = \frac{360 - 120 - 100}{120 + 100} = \frac{140}{220}$$

$$\theta_1 = 50.48°$$

$$\cos\theta_2 = \frac{BD}{AB} = \frac{360 - 120 - 80}{120 + 80} = \frac{160}{200}$$

$$\theta_2 = 36.87°$$

Consider the free body diagram of cylinder C.

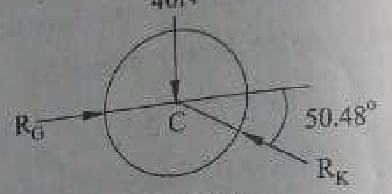


Fig. 1.103

For $\sum F_V = 0$, $R_K \sin 50.48 - 40 = 0$

$R_K = 51.48$ N

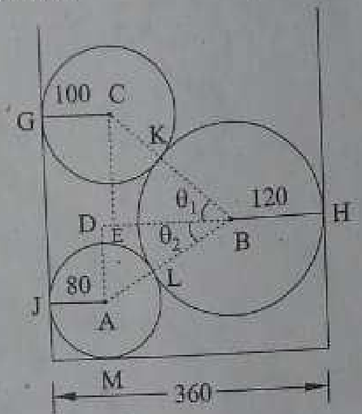


Fig. 1.102

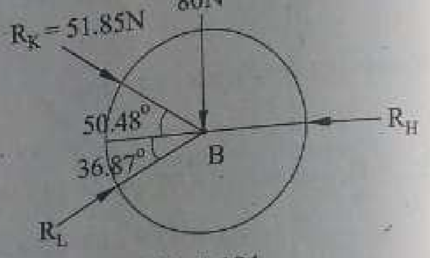


Fig. 1.104

Consider the free body diagram of cylinder B.

For $\sum F_V = 0$,

$$R_L \sin 36.87 - 80 - 51.85 \sin 50.48 = 0$$

$$R_L = \frac{51.85 \sin 50.48 + 80}{\sin 36.87}$$

$$= 200 \text{ N}$$

Consider the free body diagram of cylinder A.

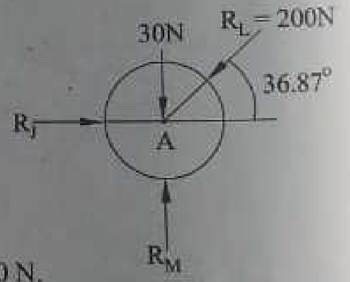


Fig. 1.105

For $\sum F_H = 0$,

$$R_J - 200 \cos 36.87 = 0$$

$$R_J = 160 \text{ N}$$

Reaction between the cylinder A and the vertical wall is 160 N.

1.16. Method of moments

Many problems of statics can be solved by taking moments of forces acting on the body about a convenient axis or point. The effectiveness of a force as regards to its tendency to produce rotation of a body about an axis or a point is called moment of the force. It is measured by the product of the magnitude of the force and arm of the force. Arm of force, also called moment arm is the perpendicular distance of the line of action of the force from the moment centre. Moment centre is the point about which moment of a force is measured.

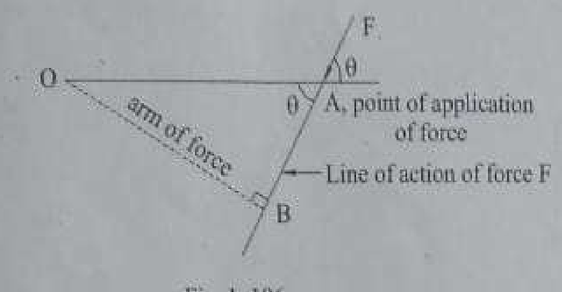


Fig. 1. 106

The line perpendicular to the plane containing the force and passing through the moment centre is called axis of the moment. When the force is expressed in newtons (N) and the arm of force in metres then the moment of force will be expressed in newton-metre (Nm). The moment of a force has not only a magnitude but also a sense. Depending upon the relative position of the force and moment centre, the moment of a force will be either clockwise or counter clockwise. The sense of moment of a force is taken as clockwise when the force rotates or tends to rotate the arm of force in clockwise direction about the moment centre. Similarly the sense of moment of a force is taken as counter clockwise when the force rotates or tends to rotate the arm of force in counterclockwise direction about the moment centre. If a clockwise moment is taken as positive then the counter clockwise moment should be taken as negative.

O is the moment centre, A is the point of application of force F, OB is the arm of force. θ is the inclination of force F with horizontal. Moment of the force F about O.

$$M_O = F \times OB = F \times OA \sin\theta.$$

Moment of a force will be maximum when the line of action of the force is perpendicular to the line joining the moment centre and point of application of the force. Moment of a force will be zero when, (i) the force acts at the moment centre itself and (ii) when the line of action of the force passes through the moment centre.

To calculate the moment of a force,

1. Identify the force.
2. Identify the moment centre.
3. Identify the line of action of the force.
4. Calculatle the perpendicular distance of the line of action of the force from the moment centre.
5. Multiply the force by the calculated perpendicular distance.

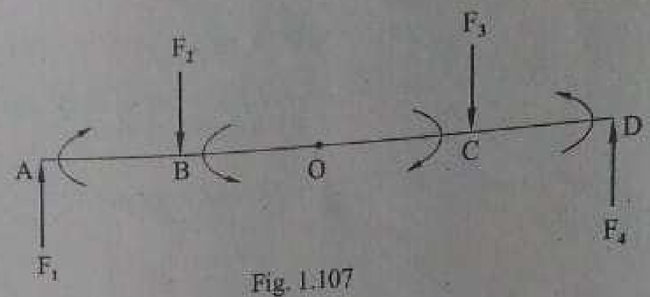


Fig. 1.107

Moment of force F_1 about O is $F_1 \times$ OA, C.W.

Moment of force F_2 about O is $F_2 \times$ OB, C.C.W.

Moment of force F_3 about O is $F_3 \times$ OC, C.W.

Moment of force F_4 about O is $F_4 \times$ OD, C, C.W.

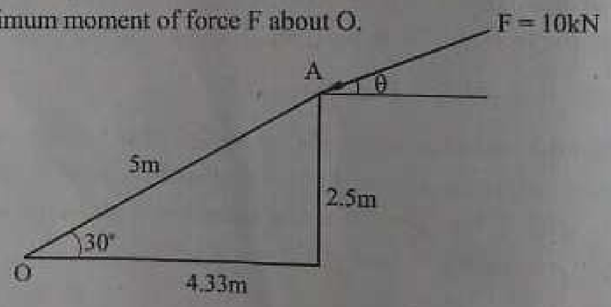
Example 1.29.

Calculate the moment of the force F = 10 kN acting at point A as shown in Fig 1.108. when the angle, (i) $\theta = 0$

(ii) $\theta = 30^0$

(iii) $\theta = 90^0$

Also calculate the maximum moment of force F about O.



Solution

Fig. 1.108

(i) When θ = 0, the line of action is horizontal and hence the arm of force is 2.5 m.

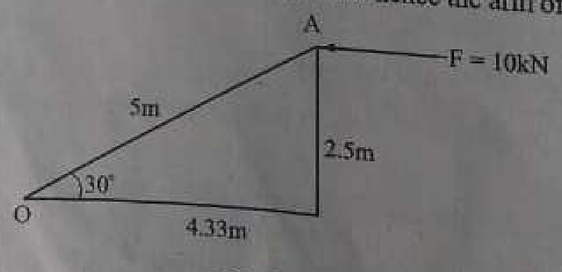


Fig. 1.109

$$M_0 = 10 \times 2.5 = 25 \text{ k N m (c.c.w)}$$

(ii) When $\theta = 30^0$, the line of action of force F passes through the moment centre.

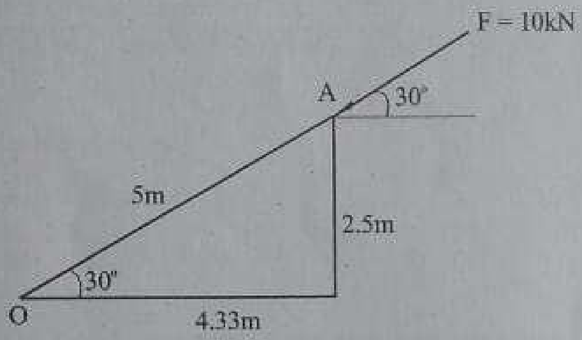


Fig. 1.110

Therefore $M_0 = 10 \times 0 = 0$

(iii) When $\theta = 90^0$, the line of action is vertical and hence the arm of force is 4.33 m.

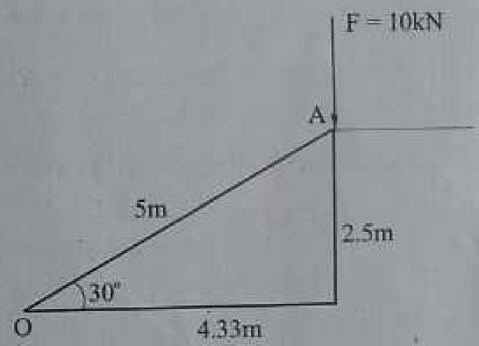
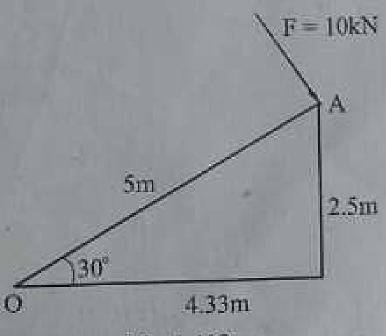


Fig. 1.111

$$M_0 = 10 \times 4.33 = 43.3 \text{ k Nm (c.w)}$$

The moment of force will be maximum when it is perpendicular to OA.



$M_0 = 10 \times 5 = 50$ kNm.

Fig. 1.112

Example 1.30

A uniform wheel 60 cm diameter weighing 1000 N rests against a rectangular obstacle 15 cm height as shown in Fig. 1.113. Find the least force required, which when acting through the centre of the wheel will just turn the wheel over the corner A of the block. Also find the reaction of the block.

Solution

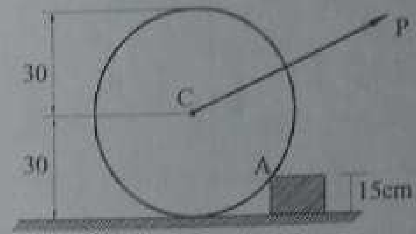


Fig. 1.113

When the roller is just turned about A, the contact at B breaks and hence there is no reaction at B. Let P be the applied force and R_A be the reaction at the contact point A.

$$BC = AC\cos\theta + 150$$

$$300 = 300\cos\theta + 150$$

$\theta = 60^\circ$, the inclination of line joining A and C with vertical, $\theta = 60^\circ$.

Let the inclination of the force P with AC be α.

Taking moments of P and W about A,

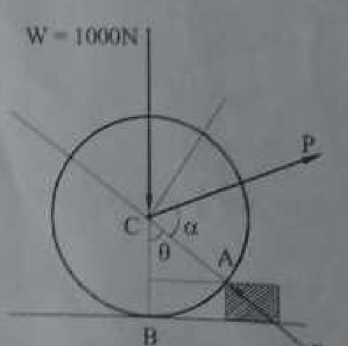


Fig. 1.114

$$\text{For } \sum M = 0,$$

$$(P\sin\alpha)\times AC - W\times(AC\sin\theta) = 0$$

$$P\sin\alpha = W\sin 60 = 1000\sin 60$$

$$= 866\text{ N.}$$

$$P = \frac{866}{\sin \alpha}$$

For P to be minimum, sin α should be maximum. For this α should be 90^0

$$P = 866 \text{ N}$$

Resolving the forces along AC,

$$R_A - P \cos \alpha - W \cos \theta = 0$$

$$R_A = 866 \cos 90 + 1000 \cos 60$$

$$= 500 \text{ N}$$

1.17. Varignon's theorem of moments.

French mathematician Varignon (1654 - 1722) gave the following theorem which is also known as principle of moments. Varignon's theorem states that the moment of a force about any axis is equal to the sum of moments of its components about that axis.

Consider a force F acting at a point A. F_1 and F_2 are the components of F along any two directions. The moment of F about an axis through an arbitrary point O is F x d, where d is the arm of force F. d_1 and d_2 are the arm of forces of F_1 and F_2 respectively. Sum of moments of the components F_1 and F_2 about O is $F_1d_1 + F_2d_2$. It is to be proved that $F \times d = F_1 \times d_1 + F_2 \times d_2$.

Join A and O. Draw a line through A and perpendicular to OA. Let AG, AJ and AE be the rectangular components of F_1, F_2 and F respectively.

Referring Fig 1.115

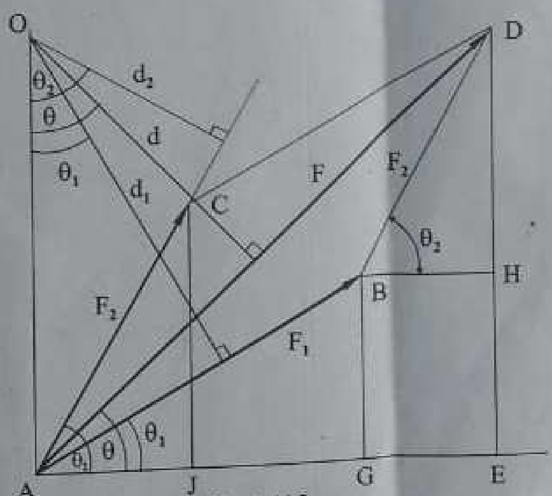


Fig. 1.115

$$AE = AG + GE$$
$$= AG + BH$$
$$= AG + BD \cos \theta_2$$
$$= AG + AC \cos \theta_2$$
$$AD \cos \theta = AB \cos \theta_1 + AC \cos \theta_2$$
$$F \cos \theta = F_1 \cos \theta_1 + F_2 \cos \theta_2 \text{. (Theorem of resolved parts)}$$

Multiplying by OA.

$$F \times OA \cos \theta = F_1 \times OA \cos \theta_1 + F_2 \times OA \cos \theta_2$$
$$F \times d = F_1 \times d_1 + F_2 \times d_2$$

Moment of F about O = moment of F_1 about O + moment of F_2 about O.

Since F_1 and F_2 are the components of the force F, and O is an arbitrary point, moment of a given force about any point = sum of moments of its components about the same point.

Example 1.31.

Calculate the moment of the given force F = 10 kN about point O, as shown in Fig. 1.116.

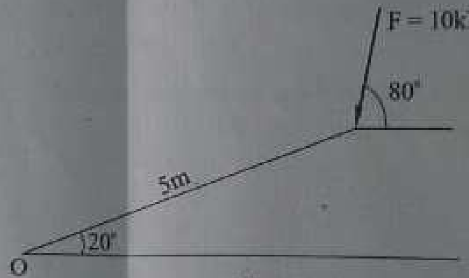


Fig. 1.116

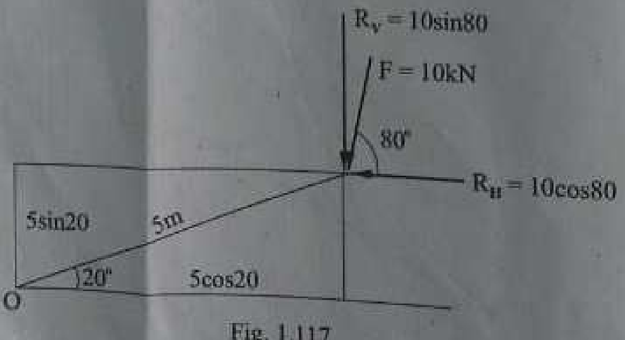


Fig. 1.117

Taking moments of horizontal and vertical components of the given force,

$$M_o = (10 \sin 80) \times (5 \cos 20) - (10 \cos 80) \times (5 \sin 20)$$

$$= 43.30 \text{ kN m.}$$

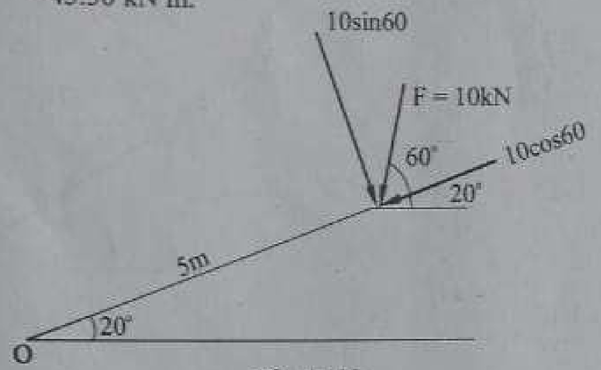


Fig. 1.118

Taking moments of components of the given force along OA and perpendicular to OA,

$$M_o = (10 \cos 60) \times 0 + (10 \sin 60) \times 5 = 43.30 \text{ kNm.}$$

Example 1.32

Calculate the moment of the force system shown in Fig 1.119 about O, using Varignon's principle.

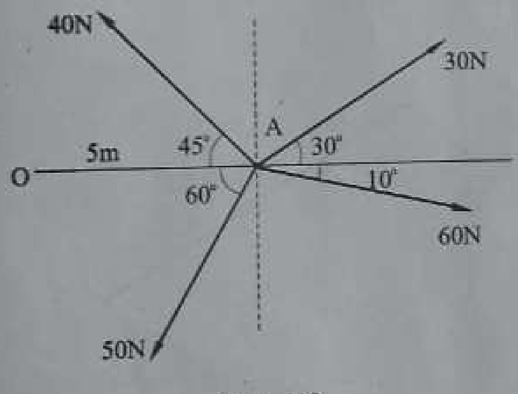


Fig. 1.119

$$\Sigma F_x = 30 \cos 30 + 60 \cos 10 - 40 \cos 45 - 50 \cos 60 = 31.78 \text{ N.}$$

$$\Sigma F_y = 30 \sin 30 + 40 \sin 45 - 50 \sin 60 - 60 \sin 10$$

$$= -10.44 \text{ N}$$

$$\text{Resultant, } R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(31.78)^2 + (-10.44)^2}$$

$$= 33.45 \text{ N}$$

$$\tan\theta = \left|\frac{\sum F_y}{\sum F_x}\right|$$

$$\theta = \tan^{-1}\left|\frac{\sum F_y}{\sum F_x}\right|$$

$$= \tan^{-1}\left|\frac{10.44}{31.78}\right| = 18.19^0$$

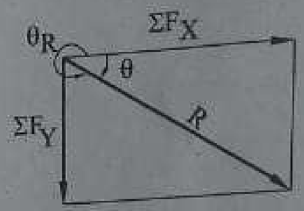


Fig. 1.120

Since the resultant is in the fourth quadrant, the inclination of the resultant,

$\theta_R = 360 - \theta = 360 - 18.19 = 341.81^0$

$M_o = R \sin\theta \times 5$

$$= 33.45 \times (\sin 18.19) \times 5$$

$$= 33.45 \times (\sin 18.19) \times 5$$

$$= 52.21 \text{ Nm}$$

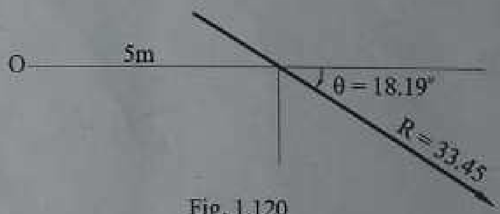


Fig. 1.120

Example 1.33

Forces of 15N, 20N, 25N, 35N and 45N act at an angular point of a regular hexagon towards the other angular points as shown in Fig.1.121. Calculate the moment of these forces about A, using Varignon's principle.

Solution.

Resolving the forces along X axis,

$$\sum F_x = 15 + 20\cos 30 + 25\cos 60 + 0 - 45\cos 60 = 22.32 \text{ N}$$

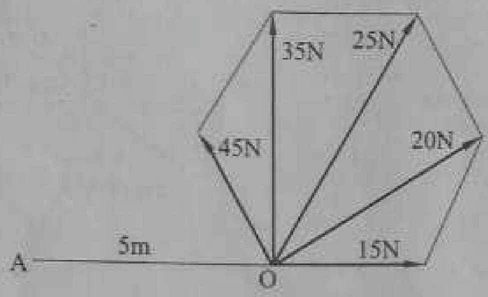


Fig. 1.121

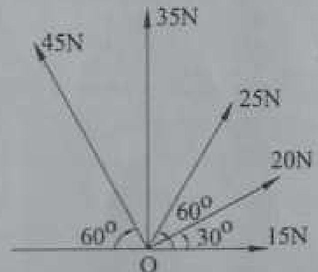


Fig.1.122

Resolving the forces along y axis,

$$\Sigma F_y = 0 + 20 \sin 30 + 25 \sin 60 + 35 + 45 \sin 60 = 105.62 \text{ N}$$

$$\text{Resultant, } R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$= \sqrt{22.32^2 + 105.62^2}$$

$$= 107.95 \text{ N}$$

Inclination of resultant with horizontal,

$$\theta = \tan^{-1} \frac{|\Sigma F_y|}{|\Sigma F_x|}$$

$$= \tan^{-1} \frac{105.62}{22.32}$$

$$= 78.07^\circ$$

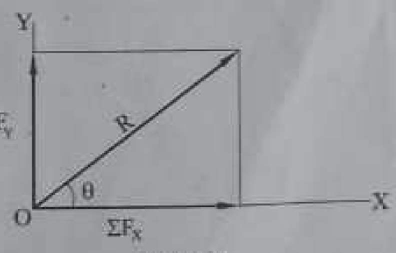


Fig. 1.123

According to Verignon's principle, sum of moment of the forces about A is equal to the moment of their resultant about A.

Moment of resultant R about A,

$M_A = R \sin\theta \times 5$

$= 107.95 \times \sin 78.07 \times 5$

$= 528.09\text{Nm}.$

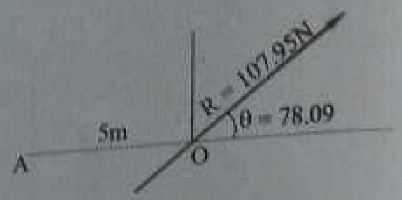


Fig. 1.124